

## Review Exercises

1. Let  $A$  and  $B$  be two events defined on a sample space  $S$  such that  $P(A) = 0.3$ ,  $P(B) = 0.5$ , and  $P(A \cup B) = 0.7$ .

- (a) Find  $P(A \cap B)$ .
- (b) Find  $P(A^C \cup B^C)$ .
- (c) Find  $P(A^C \cap B)$ .

2. In a newly released martial arts film, the actress playing the lead role has a stunt double who handles all of the physically dangerous action scenes. According to the script, the actress appears in 40% of the film's scenes, her double appears in 30%, and the two of them are together 5% of the time.

- (a) What is the probability that in a given scene only the stunt double appears?
- (b) What is the probability that neither the lead actress nor the double appears?

3. Show that:

$$f(x) = \frac{1}{1+\lambda} \left( \frac{\lambda}{1+\lambda} \right)^x$$

for  $x = 0, 1, 2, \dots$  and  $\lambda > 0$  is a valid probability mass function.

4. A criminal court judge has heard many cases where the defendant was charged with grand theft auto and the jury returned a guilty verdict. But the final disposition was not always the same. Mitigating circumstances of various kinds led the judge over the years to impose unequal jail terms for what was basically the same offense. Looking back over the court transcripts, he sees that  $y$ , the imposed sentence length (in years) has a distribution that can be described quite well by a continuous probability function having the form:

$$f(y) = \frac{1}{9}y^2 \text{ for } 0 < y < 3.$$

What proportion of those found guilty spent less than a year in jail?

5. The reaction time of motorists over the age of 70 to a certain visual stimulus is described by a continuous probability function of the form:

$$f(y) = ye^{-y} \text{ for } y > 0$$

where  $y$  is measured in seconds. What is the probability that a randomly selected "old" motorist requires longer than 1.5 seconds to react?

6. A card is drawn from a poker deck. What is the probability that the card is a club, given that the card is a king?
7. The number of bold-faced lies,  $x$ , that a certain politician tells during a news conference is described by the probability mass function:

$$f(x) = \frac{8}{15} \left( \frac{1}{2} \right)^x \text{ for } x = 0, 1, 2, 3.$$

What is the probability that he tells at least two lies given that he tells at least one?

8. A 12 pack of beer contains 5 Rolling Rocks, 4 Yuenglings, and 3 Keystone Lights. Four bottles of beer are randomly, sequentially, and without replacement. What is the probability of obtaining the sequence (RR, Y, RR, KL)?
9. Urn I contains two red chips and four white chips. Urn II, three red and one white. A chip is drawn at random from urn I and transferred to urn II. Then a chip is drawn from urn II. What is the probability that the chip drawn from urn II is red?

10. During a power blackout, 100 persons are arrested on suspicion of looting. Each is given a polygraph test. From past experience it is known that the polygraph is 90% reliable when administered to a guilty suspect and 98% reliable when given to someone who is innocent. Suppose that of the 100 persons taken into custody, only 12 were actually involved in any wrongdoing. What is the probability that a given suspect is innocent given that the polygraph says he is guilty?
11. Suppose that  $P(A \cap B) = 0.2$ ,  $P(A) = 0.6$ , and  $P(B) = 0.5$ .
  - (a) Are A and B mutually exclusive?
  - (b) Are A and B independent?
  - (c) Find  $P(A^C \cup B^C)$ .
12. In the wake of the *Challenger* disaster, NASA officials estimated that the chances of a similar catastrophic accident during a shuttle flight are roughly 1 in 78. If that assessment is correct, what is the probability that there will be a least one crash among the next 20 launches? Assume that the outcome of any flight has no effect on the success of any other flights.
13. Four men and four women are to be seated in a row of chairs numbered 1 through 8.
  - (a) How many total arrangements are possible?
  - (b) How many arrangements are possible if the men are required to sit in alternate chairs?
14. A restaurant offers a choice of 4 appetizers, 14 entrees, 6 desserts, and 5 beverages. How many different meals are possible if a diner intends to order only 3 courses? (Consider a beverage to be a "course.")
15. Which state name can generate more permutations, TENNESSEE or FLORIDA?
16. How many numbers greater than 4,000,000 can be formed from the digits 2, 3, 4, 4, 5, 5, 5?
17. The crew of *Apollo 17* consisted of two pilots and one geologist. Suppose that NASA had actually trained a total of nine pilots and four geologists. How many possible *Apollo 17* crews could have been formed?
18. Ten basketball players meet in the school gym for a pickup game. How many ways can they form two teams of five each?
19. Does a monkey have a better chance of rearranging ACCLLUUS to spell CALCULUS, or AABEGLR to spell ALGEBRA?
20. Five fair dice are rolled. What is the probability that the faces showing constitute a "full house" – that is, three faces show one number and two faces show a second number?
21. A bridge hand (13 cards) is dealt from a standard 52-card deck. Let A be the event that the hand contains four aces; let B be the event that the hand contains four kings. Find  $P(A \cup B)$ .
22. For one-pair poker hands, why is the number of denominations for the three single cards  $\binom{12}{3}$  rather than  $\binom{12}{1}\binom{11}{1}\binom{10}{1}$ ?
23. A hung jury is one that is unable to reach a unanimous decision. Suppose that a pool of 25 potential jurors are assigned to a murder case where the evidence is so overwhelmingly against the defendant that 23 of the 25 would return a guilty verdict. The other two potential jurors would vote to acquit regardless of the facts. What is the probability that a 12-member panel chosen at random from the pool of 25 will be unable to reach a unanimous decision?
24. The great English diarist Samuel Pepys asked his friend Sir Isaac Newton the following question: Is it more likely to get at least one 6 when 6 dice are rolled, at least two 6's when 12 dice are rolled, or at least three 6's when 18 dice are rolled? After considerable correspondence, Newton convinced the skeptical Pepys that the first event is the most likely. Compute the three probabilities.

25. Suppose that the variation in a measurement  $Y$  is described by the continuous p.d.f.:

$$f_Y(y) = 6y(1 - y)$$

for  $0 \leq y \leq 1$ . If five independent values of  $Y$  are observed, what is the probability that exactly three will lie between  $\frac{1}{4}$  and  $\frac{1}{2}$ ?

26. Find the cumulative distribution function  $F_Y(y)$  of the random variable whose p.d.f is the function:

$$f_Y(y) = \begin{cases} 3y(1 - y) & \text{for } 0 \leq y \leq 1 \\ \frac{1}{2} & \text{for } 2 \leq y \leq 3 \end{cases} .$$

27. Let  $X$  be a binomial random variable based on  $n$  trials and a success probability of  $p_X$ . Let  $Y$  be an independent binomial random variable based on  $m$  trials and a success probability of  $p_Y$ . Find the expected value and variance of  $W$ , where  $W = 4X + 6Y$ .

28. Find the expected value of  $e^{3X}$  if  $X$  is a binomial random variable with  $n = 10$  and  $p = \frac{1}{3}$ .

29. Suppose that 1% of all items in a supermarket are not priced properly. A customer buys 10 items. What is the probability that she will be delayed by the cashier because one or more of her items requires a price check?

30. Suppose that typographical errors are made at the rate of 0.4 per page in State Tech's campus newspaper. If next Tuesday's edition is 16 pages long, what is the probability that fewer than 3 typos will appear?

31. Mensa (from the Latin word for "mind") is an international society devoted to intellectual pursuits. Any person who has an IQ in the upper 2% of the general population is eligible to join. Assuming that IQs are normally distributed with mean 100 and standard deviation 16, what is the lowest IQ that will qualify a person for membership?

32. The following letter was written to Dear Abby in 1973: *Dear Abby: You wrote in your column that a woman is pregnant for 266 days. Who said so? I carried my baby for 10 months and 5 days, and there is no doubt about it because I know the exact date that my baby was conceived. My husband is in the Navy and it couldn't have possibly been conceived any other time because I saw him only once for an hour, and I didn't see him again until the day before the baby was born. I don't drink or run around, and there is no way this baby isn't his, so please print a retraction about the 266-day carrying time because otherwise I am in a lot of trouble. San Diego Reader.* Whether or not San Diego Reader is telling the truth is a judgment that lies beyond the scope of any statistical analysis, but quantifying the plausibility of her story does not. According to the collective experience of generations of pediatricians, pregnancy durations,  $Y$ , tend to be normally distributed with a mean of 266 days and a standard deviation of 16 days. Do a probability calculation that addresses San Diego Reader's credibility. What would you conclude?

33. At State University, the average score of the entering class on the verbal portion of the SAT is 565, with a standard deviation of 75. Marian scored a 660. How many of the State's other 4250 freshmen did better? Assume that the scores are normally distributed.

34. It is estimated that 80% of all 18-year-old women have weights ranging from 103.5 to 144.5 pounds. Assuming the weight distribution can be adequately modeled by a normal curve and assuming that 103.5 and 144.5 are equidistant from the average weight  $\mu$ , calculate  $\sigma$ .

35. Because of her past convictions for mail fraud and forgery, Jody has a 30% chance each year of having her tax returns audited. What is the probability that she will escape detection for at least three years? Assume that she exaggerates, distorts, misrepresents, lies and cheats every year.

36. An underground military installation is fortified to the extent that it can withstand up to three direct hits from air-to-surface missiles and still function. Suppose an enemy aircraft is armed with missiles, each having a 30% chance of scoring a direct hit. What is the probability that the installation will be destroyed with the seventh missile fired?

37. Suppose that the time (in hours) taken by a homeowner to mow his lawn is a r.v.  $X$  having a gamma distribution with parameters  $\alpha = 2$  and  $\theta = \frac{1}{2}$ . What is the probability that it takes at most 1 hour to mow the lawn?
38. The time (in seconds) that it takes a librarian to locate a card in a file of records on checked-out books has an exponential distribution with expected time equaling 20 seconds. What is the probability that a randomly selected librarian takes more than 20 seconds to locate a card?
39. The length of time  $Y$  (in years) that a malaria patient spends in remission is presumed to follow the p.d.f.:

$$f_Y(y) = \frac{1}{9}y^2 \text{ for } 0 < y < 3$$

What is the average length of time such a patient spends in remission?

40. Suppose that  $X$  and  $Y$  have the joint p.m.f.:

$$f(x, y) = \frac{xy^2}{39}$$

for the points (1,2), (1,3), (2,2), and (2,3), and is 0, otherwise.

- (a) Find the conditional probability that  $X$  is 1 given that  $Y$  is 2.  
 (b) Find  $E(X|Y = 2)$ .

41. Given the joint p.d.f.

$$f(x, y) = 2e^{-(x+y)}$$

for  $0 < x < y$  and  $y > 0$ .

- (a) Find  $P(Y < 1|X < 1)$ .  
 (b) Find  $h(y|x)$ .

42. Two chips are drawn at random and without replacement from an urn that contains five chips, numbered 1 through 5. If the sum of the chips is even, the random variable  $X$  equals 5; if the sum of the chips drawn is odd,  $X = -3$ . Find the moment-generating function for  $X$ .