EDGEWORTH EXPANSIONS: A BRIEF REVIEW OF ZHIDONG BAI'S CONTRIBUTIONS

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Professor Bai's contributions to Edgeworth Expansions are reviewed. Author's collaborations with Professor Bai on the topic are also discussed.

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I have the pleasure of collaborating with Professor Bai Zhidong on many papers including three [2–4] on Edgeworth expansions.

The earliest work of Bai on Edgeworth expansions that I came across is the English translation [10] of his joint work with Lin Cheng, which was first published in Chinese. They investigate expansions for the distribution of sums of independent but not necessarily identically distributed random variables. The expansions are obtained in terms of truncated moments and characteristic functions. From this, they derive an ideal result for non-uniform estimates of the residual term in the expansion. In addition they also derive the non-uniform rate of the asymptotic normality of the distribution of the sum of independent but identically distributed random variables, extending some of the earlier work by A. Bikyalis [13] and L. V. Osipov [17]. Few years later Bai [7] obtains Edgeworth expansions for convolutions by providing bounds for the approximation of $\psi * F_n$ by $\psi * U_{kn}$, where F_n denotes the distribution function of the sum of n independent random variables, ψ is a function of bounded variation and U_{kn} denotes the "formal" Edgeworth expansion of F_n up to the kth order.

Many important statistics can be written as functions of sample means of random vectors. Bhattacharya and Ghosh [11] made fundamental contributions to the theory of Edgeworth expansions for functions of sample means of random vectors. Their results are derived under Cramér's condition on the joint distribution of all the components of the vector variable. However, in many practical situations, such as ratio statistics [6] and survival analysis, only one or a few of the components satisfy Cramér's condition while the rest do not. Bai along with Rao [8] established Edgeworth expansions for functions of sample means when only the partial Cramér's condition is satisfied. Bai & Rao [9] derived Edgeworth expansions on ratio's of sample means, where one of the variables is counting (lattice) variable. Such ratios arise in survival analysis in measuring and comparing the risks of exposure of individuals to hazardous environments. Bai in collaboration with Babu [3] has developed Edgeworth expansions under a partial Cramér's condition, extending the results of Bai & Rao [8, 9]. But the results of [6, 8] require moments higher than the ones appearing in the expansions. However, in [3], the conditions on the moments are relaxed to the minimum needed to define the expansions. The results generalize Hall's [16] work on expansions for student's t-statistic under minimal moment conditions, and partially some of the derivations of [12, 14, 15].

In the simple errors-in-variables models, a pair (X_i, Y_i) of attributes are measured on the i-th individual with error (δ_i, ϵ_i), where $E(\delta_i) = E(\epsilon_i) = 0$, and $X_i - \delta_i$ and $Y_i - \epsilon_i$ are related by a linear equation. That is, $X_i = v_{in} + \delta_i$ and $Y_i = \omega + \beta v_{in} + \epsilon_i$, where v_{in} are unknown nuisance parameters. Various estimators of the slope parameter β are derived by Bai & Babu [2] under additional assumptions. Even though the residuals in these errors-in-variables models are assumed to be independent and identically distributed random variables, the statistics of interest turn out to be functions of means of independent, but not identically distributed, random vectors. They also demonstrate that the bootstrap approximations of the sampling distributions of these estimators correct for the skewness. The bootstrap distributions are shown to approximate the sampling distributions of the studentized estimators better than the classical normal approximation.

Babu & Bai [4] take the results of Babu & Singh [5], on Edgeworth expansions for statistics based on samples from finite populations, to a new direction by developing mixtures of global and local Edgeworth expansion for functions of random vectors. Edgeworth expansions are obtained for

$$P\Big\{\sum_{j=1}^{N} a_{j,N}(Z_j - E(Z_j)) \in H, \sum_{j=1}^{N} Z_j = n\Big\}$$

as a combination of global and local expansions, where $\{Z_j\}$ is an i.i.d. sequence of random variables with a lattice distribution and $\{a_{j,N}\}$, is an array of constants. From this, expansions for conditional probabilities

$$P\Big\{\sum_{j=1}^{N} a_{j,N}(Z_j - E(Z_j)) \in H \mid \sum_{j=1}^{N} Z_j = n\Big\}$$

are derived using local expansions for $P\{\sum_{j=1}^{N} Z_j = n\}$. In the case of absolutely continuous Z_1, the expansions are derived for $(\sum_{j=1}^{N} a_{j,N} Z_j)/(\sum_{j=1}^{N} Z_j)$. These results are then applied to obtain Edgeworth expansions for bootstrap distributions, for Bayesian bootstrap distributions, and for the distribution of statistics based on samples from finite populations. The Bayesian bootstrap is shown to be second-order correct for smooth positive '*priors*', whenever the third cumulant of the '*prior*' is equal to the third power of its standard deviation. As a consequence, it is easy to conclude that among the standard gamma '*priors*', the only one that leads to second order correctness is the one with mean 4. Similar results are established for the weighted bootstrap when the weights are constructed from random variable with a lattice distribution.

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