

A statistical model for the relation between exoplanets and their host stars

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Abstract: A general model is proposed to explain the relation between the extrasolar planets (or exoplanets) detected until June 2008 and the main characteristics of their host stars through statistical techniques. The main goal is to establish a mathematical relation among the set of variables which better describe the physical characteristics of the host star and the planet itself. The host star is characterized by its distance, age, effective temperature, mass, metallicity, radius and magnitude. The exoplanet is described through its physical parameters (radius and mass) and its orbital parameters (distance, period, eccentricity, inclination and major semiaxis). As a first approach we consider that only the mass of the exoplanet is being determined by the physical properties of its host star. The proposed model is then validated through statistical analysis.

Keywords: Linear regression, Cross-sectional data, Exoplanets

1 Introduction

An extrasolar planet (or exoplanet) is a planet which orbits a star other than the Sun, and therefore belongs to a planetary system other than our Solar System. The first extrasolar planet around a main sequence star was discovered in 1995 (Mayor and Queloz, 1995). Actually more than 300 exoplanets have been documented and most of them with masses greater than Jupiter's mass (Schneider, 2009). Detecting an exoplanet is a very difficult task because they do not emit any electromagnetic radiation of their own and are completely obscured by their extremely bright host stars, that is, normal telescope observation techniques cannot be used. Thus, in order to find exoplanets, a variety of techniques like the radial velocity, pulsar timing, astrometry, gravitational lensing, spectrometry and photometry (De Pater and Lissauer, 2001) are used. The main purpose of any method is to detect the effect produced by the exoplanet on its stellar system. Besides the discoveries it is important to search for models that can explain the origin, formation and possible migration of these bodies. For example,

Rice and Armitage (2005) have investigated how the statistical distribution of extrasolar planets may be combined with knowledge of the host stars' metallicity to yield constraints on the migration histories of gas giant planets. Moreover in a series of papers (Udry et al., 2003; Santos et al., 2003; Eggenberger et al., 2004; Halbwegs et al., 2005) the emerging properties of planet-host stars and characteristics of the different orbital-element distributions of exoplanetary systems have been studied. In this work we analyze the cross-sectional data for the exoplanets detected until June 2008 through linear regression techniques. The purpose of this kind of analysis is to verify the relation between the host star and its orbiting planet. For example, if the planet's mass is strongly determined by the type of star and hence affects the planetary formation stage.

1.1 Characteristics of the data catalog: Stars and Planets

The catalog was created in February 1995 to facilitate the progress of the new field named Exoplanetology through the publication of recent detections and their associated data. The catalog is interactive and it is available in the webpage: <http://exoplanet.eu>.

Until June 2008 the catalog contains: 303 exoplanets and 259 planetary systems (31 multiple systems). Two important considerations are: 1) the mass of the exoplanet is -at least- $13 M_J$ (Jupiter's mass) and 2) the data source must be reliable, that is, previously published in referred journals, presented in conferences, among others.

- **Stars:** The stellar data are taken from well-known databases like Simbad or directly from published papers. The basic physical characteristics of a star are: radial velocity, mass, metallicity, age and distance.
- **Planets:** These data are taken from published papers and from the sites: Anglo-Australian Planet Search; California and Carnegie Planet Search; Geneva Extrasolar Planet Search Programmes; Transatlantic Exoplanet Survey and the Department of Astronomy at University of Texas.

2 The General Model: Multiple Regression Analysis

We start with the following model (Model A) described by the equation:

$$M_P = \alpha_1 + \alpha_2 DS + \alpha_3 AS + \alpha_4 TS + \alpha_5 MS + \alpha_6 METAL + \alpha_7 MAG + \alpha_8 RS + u_i \quad (1)$$

where M_P is the exoplanet's mass and α_i are the coefficients for each term. Eq. (1) expresses the exoplanet's mass M_P in terms of the values of the

TABLE 1. Estimated values for the parameters.

Model A				
Variable	α_i	Standard Error	t-statistic	Probability
<i>C</i>	-9.1773	5.0051	-1.8336	0.0685
<i>DS</i>	-0.0113	0.0074	-1.5215	0.1301
<i>ES</i>	0.0288	0.0872	0.3299	0.7419
<i>TS</i>	0.0013	0.0007	1.9169	0.0570
<i>MS</i>	1.9385	1.3721	1.4128	0.1596
<i>METAL</i>	-1.7493	1.2586	-1.3899	0.1664
<i>MAG</i>	0.3689	0.2850	1.2944	0.1973
<i>RS</i>	0.2335	0.1183	1.9747	0.0499
Model B				
Variable	α_i	Standard Error	t-statistic	Probability
<i>C</i>	-2.5169	1.0840	-2.3218	0.0213
<i>ES</i>	-0.0493	0.0321	-1.5345	0.1265
<i>TS</i>	0.0003	0.0002	1.7417	0.0831
<i>MS</i>	1.1772	0.3964	2.9698	0.0033
<i>METAL</i>	-1.1370	0.5001	-2.2738	0.0241
<i>SIST*MS</i>	-0.1809	0.2188	-0.8269	0.4093
<i>SIST*METAL</i>	0.5629	0.8338	0.6752	0.5003

variables representing the features of the host star. This set of variables contains: the distance, *DS*; the age, *AS*; the temperature, *TS*; the mass, *MS*; the metallicity, *METAL*; the magnitude, *MAG* and the radius, *RS*. Finally u_i are the random errors.

We estimate the unknown parameters in Eq. (1) by Ordinary Least Squares (OLS). The results are shown in Table 1 where we also include the values of the t-statistics and their associated probabilities for the coefficient significance tests. From the estimated values we conclude that the only significant variable for the Model A is *RS*.

2.1 Verification of the linear regression assumptions (Model A)

1. Linearity: The model passed all the Ramsey tests for linearity. We conclude that the proposed functional form is adequate.
2. Omitted Variables: According to the star formation theory, the variables *MS* and *METAL* must be included to explain the relation between the mass of the exoplanet and its host star.
3. Multicollineality: There is a possible weak correlation between *MAG* and *DS*.

4 A statistical model for exoplanets

4. Heteroskedasticity: From the White test on the residuals we conclude that they are not heteroskedastic, that means the residuals are homoskedastic.
5. Normality: From the value of the Jarque-Bera statistic we conclude that the residuals are not normally distributed.
6. Homogeneity: Defining the "dummy" variable as $SIST$ (0 means that the exoplanet belongs to a single planetary system and 1 refers to a multiple planetary system) we conclude that the model is homogeneous.

Statistical model must satisfy all the assumptions mentioned above to be correctly specified. In our case, the Model A needs some modifications, for example, another functional form and/or the consideration of an adequate "dummy" variable. In such a case we derive the Model B:

$$\log(M_P) = \alpha_1 + \alpha_2 ES + \alpha_3 TS + \alpha_4 MS + \alpha_5 METAL + \gamma_1 SMS + \gamma_2 SMET + u_i \quad (2)$$

where $SMS = SIST * MS$ and $SMET = SIST * METAL$ are two new variables that take into account the fact that the exoplanet can belong to a single or a multiple planetary system. The parameters are estimated through OLS and the results are summarized in Table 1.

2.2 Verification of the linear regression assumptions (Model B)

1. Linearity: The model passed all the Ramsey tests for linearity. Moreover we conclude that the new functional form is more adequate than the presented in Model A.
2. Omitted Variables: The tests indicate that the variables ES and TS must be excluded. However, under this situation the linearity is not preserved and we lose important physical information about the host star.
3. Multicollinearity: There is no correlation among the independent variables.
4. Heteroskedasticity: From the White test on the residuals we conclude that they are heteroskedastic.
5. Normality: From the value of the Jarque-Bera statistic we conclude that the residuals are normally distributed.
6. Homogeneity: The model has already included the effect of a dummy variable.

Model B (*log-linear*) is slightly better than Model A in the sense that we have improved some of the discrepancies previously detected in the basic assumptions. However, this latter model cannot be considered yet to explain the relation between an exoplanet and its host star. Including the effect of a "dummy" variable seems to be a clue for another type of model. This binary behavior is discussed in the next section.

3 Multiple Regression Analysis with Binary Dependent Variables: a different approach

Based on the data, the dependent variable (exoplanet) is simultaneously determined by several parameters, qualitative and quantitative. In this work we have just assumed that the mass, M_P , is the quantitative variable that represents the whole physical/orbital characteristics of the planet. However, this fact is not completely true and more qualitative information must be taken into account for the model.

In the context of the variable *exoplanet*, the relevant information can be captured by defining a binary variable or a zero-one variable. An example of such a variable was introduced in Section 2 as *SIST* and it is related to the fact that the exoplanet can belong to a single or a multiple planetary system, in other words, $SIST = 0$ if the exoplanet belongs to a single planetary system and $SIST = 1$ in other case.

Under this new approach some binary models can be employed and their choice depends on the data distribution. For example, for a normal distribution we apply the *probit* model, for a logistic distribution we apply the *logit* model and when the data are truncated or censored we apply the *tobit* model.

Once the model is selected, its parameters can be estimated through the traditional methods like the Maximum Likelihood (ML) and Ordinary Least Squares (OLS).

A general binary model (Model C) for this case can take the form:

$$M_P = \alpha_1 + \alpha_2 ES + \alpha_3 TS + \alpha_4 MS + \alpha_5 METAL + u_i \quad (3)$$

The special case of Model C under the binary context will be discussed elsewhere. Recently a *logit* model was developed and validated by Fressin et al. (2009). In that work the authors performed a logistic regression to model the probability that a given planet is "real" (that means, observed or detected) or just simulated.

4 Summary and Conclusions

From our extensive statistical analysis we conclude that Model B is better than Model A. We have improved its specifications through the deletion of

variables like *MAG*, *DS* and *RS* and the addition of new ones that consider the possibility of finding exoplanets in single or multiple planetary systems. At the moment this is our best representation of the relation between the exoplanet and its host star and in a future work we will consider the problem by including binary variables.

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