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On a conjecture by Erdős and its extension to additive functions on the set of pairs of integers. (English. English summary)

New trends in probability and statistics, *Vol.* 4 (*Palanga*, 1996), 261–270, *VSP*, *Utrecht*, 1997.

Let f be a real-valued additive function. It was conjectured by Erdos in 1947 that a sufficient condition to ensure that f has a limiting distribution is that, for some bounded interval J, the set $A_J := \{m\colon\ f(m) \in J\}$ has a positive natural density. This was partially solved by E. M. Paul [Sankhya Ser. A 29 (1967), 279–282; MR 38#2114], who showed that it is sufficient that f has, in the interior of J, a limiting distribution which is non-uniform (in the sense that the mass of a sub-interval I of J is not a function of the Lebesgue measure of I), and by Babu [Sankhya Ser. A 39 (1977), no. 1, 1–10; MR 58#16577], who showed, among other results, that it is sufficient that A_J has density one, or, when $f \geq 0$, that $\{m\colon\ f(m) \leq c\}$ has positive upper density for some c.

Here, an extension to additive functions in two variables is studied. Recall that $h\colon \mathbf{N}^{*2} \to \mathbf{R}$ is said to be additive if $h(ab, mn) = h(a, m) + h(b, n)$ whenever $(am, bn) = 1$. One says that a set $\mathcal{A} \subset \mathbf{N}^{*2}$ has natural density $\delta = \delta(\mathcal{A})$ if $|\mathcal{A} \cap ([1, x] \times [1, y])| \sim \delta xy$ as x and y tend independently to ∞. The main result (Theorem 1) is that if $\delta\{(m, n) \in \mathbf{N}^{*2}\colon\ c < h(m, n) \leq d\}$ exists for all $(c, d] \subset (a, b]$ and defines a non-null, non-uniform measure on $(a, b]$, then h has a limiting distribution with respect to the density δ. A second theorem investigates the case when the definition of the density is altered in the sense that the variables x and y are restricted to tend to ∞ in such a way that $y/x \to \lambda$ for some fixed $\lambda > 0$.

{For the entire collection see MR 99f:11003.}

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