2004c:62072 62F12 62F40 62G10

Babu, G. Jogesh (1-PAS-S);
Rao, C. R. [Rao, Calyampudi Radhakrishna] (1-PAS-S)

Confidence limits to the distance of the true distribution from a misspecified family by bootstrap. (English. English summary)

J. Statist. Plann. Inference **115** (2003), *no.* 2, 471–478.

A bootstrap algorithm is developed for constructing joint confidence bands for the difference between a continuous distribution function H that generates a random sample $\underline{X}_n = \{X_i,\ i = 1, 2, \ldots, n\}$ and $F(\cdot;\theta_0)$, the distribution in a parametric family $F = \{F(\cdot;\theta), \theta \in \Theta\}$ which is 'closest' to H. The distance between H and F is measured by the Kullback-Leibler divergence. Let $\{\hat{\theta}_n\}$ be a sequence of estimators, such as maximum likelihood estimators, obtained from $\underline{X}_n,\ n = 1, 2, \ldots,$ which converge to θ_0. Let F_n be the empirical distribution function computed from $\underline{X}_n$ and let $\underline{X}_n^*$ be a bootstrap sample of size n obtained from $\underline{X}_n,\ n = 1, 2, \ldots.$ The main analytic result of this paper consists of giving conditions under which the following two processes converge, as $n \to \infty$, weakly to the same mean zero Gaussian process,

$$(1) \qquad Y_n(x) = \sqrt{n}(F_n(x) - F(x;\hat{\theta}_n) - (H(x) - F(x;\theta_0))),$$

$$(2) \qquad Y_n^b(x) = \sqrt{n}(F_n^*(x) - F(x;\hat{\theta}_n^*) - (F_n(x) - F(x;\theta_0))),$$

where F_n^* is the empirical distribution function obtained from $\underline{X}_n^*$ and $\hat{\theta}_n^*$ is the estimator of θ obtained from $\underline{X}_n^*$. The continuous mapping theorem then implies that the processes

$$D_n^* = \sup(|Y_n(x)^b|, -\infty < x < \infty)$$

and

$$D_n = \sup(|Y_n(x)|, -\infty < x < \infty)$$

have the same limiting distribution. Independent bootstrap samples $\{Y_n^b\}$ can then be used to construct the desired confidence bands.

Paul I. Nelson (1-KSS-S)