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Babu, G. Jogesh (3-CONCL); **Boyarsky, Abraham** (3-CONCL); **Chaubey, Yogendra P.** (3-CONCL); **Góra, Pawel** (3-CONCL)

A new statistical method for filtering and entropy estimation of a chaotic map from noisy data. (English. English summary)

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The authors consider a discrete-time chaotic dynamical system $x_i = \tau(x_{i-1})+\xi_i$ where $\tau\colon[0,1]\to[\varepsilon,1-\varepsilon]$ is a nonlinear piecewise expanding C^2 map, perturbed by an ε-small random noise ξ with zero mean, with ξ_i independent of $\tau(x_i)$, mutually independent among themselves and with the same distribution as ξ. This process admits an absolutely continuous invariant measure and is strongly mixing with the same exponential speed as τ, a fact shown in [P. Góra, Colloq. Math. **49** (1984), no. 1, 73–85; MR0774854 (86f:58135)] and in [A. Boyarsky and P. Góra, *Laws of chaos*, Birkhäuser Boston, Boston, MA, 1997; MR1461536 (99a:58102)]. Using the statistical smooth estimation method developed in [G. J. Babu, A. J. Canty and Y. P. Chaubey, J. Statist. Plann. Inference **105** (2002), no. 2, 377–392; MR1910059 (2003d:62088)] and in [G. J. Babu and Y. P. Chaubey, “Smooth estimation of a distribution and density function on a hypercube using Bernstein polynomials for dependent random vectors”, tech. rep., Dept. Math. Stat., Concordia Univ., Montreal, QC, 2003, available at www.mathstat.concordia.ca/TechReports.html], the authors present a method for filtering τ and estimating the metric entropy $h(\tau)$ from experimental data in the form of a finite time series $X^{(n)}_{\rm data} = (x_1,x_2,\dots,x_{n-1},x_n,x_{n+1})$.

Using bivariate Bernstein polynomials, smooth versions of the empirical bivariate distribution function $F_n(x,y)$ and the empirical bivariate survival function $S_n(x,y)$ are obtained, namely $\widetilde{F}_{n,m}(x,y)$ and $\widetilde{S}_{n,m}(x,y)$. Differentiating $\widetilde{F}_{n,m}(x,y)$ with respect to (x,y) yields the smooth estimator of the bivariate density function $\widetilde{f}_{n,m}(x,y)$. Considering $X^{(n)}_{\rm data}$ as a sample of the bivariate distribution $\{(X_i,Y_i) = (x_i,x_{i+1})\}_{i=1}^n$, and using the smoothed version $\widetilde{g}_{n,m}(x)$ of the density function $g(x)$ of X, for the regression function $\tau(x)=E(Y|X=x)$ the smooth estimator

$$\widetilde{\tau}_{n,m}(x)=\int_0^1 y\widetilde{f}_{n,m}(x,y)dy/\widetilde{g}_{n,m}(x)$$

is found. Convergence theorems are stated, and using Pesin’s formula, the metric entropy is estimated by

$$h(\tau) \simeq \int_0^1 \log_2 |\widetilde{\tau}'_{n,m}(x)| \widetilde{g}_{n,m}(x) dx$$

(in the absence of noise, the method described in [H. D. I. Abarbanel, *Analysis of observed chaotic data*, Springer, New York, 1996; MR1363486 (96i:58101)] and implemented in [K. M. Short, J. Comput. Phys. **104** (1993), no. 1, 162–172; MR1198227 (93h:58087)] provides an algorithm for computing metric entropy). The results can be used for filtering τ when the source data is generated by a nonlinear dynamical system (the Kalman filter is traditionally used for linear systems with noise). An example is developed and Maple 8 code is available on request. *Ricardo Gómez* (MEX-NAM-IM)