

## Practice Problems for Midterm 1

**Answers are included! If any answer happens to be wrong, please let me know.**

1. A laboratory blood test is 95 percent effective in detecting a certain disease when it is, in fact, present. However, the test also yields a false positive result for 1 percent of the healthy people tested. (That is, if a healthy person is tested, then, with probability .01, the test result will imply he or she has the disease.) If .5 percent of the population actually has the disease, what is the probability a person has the disease given that the test result is positive?  
(0.323)
2. If  $A \subset B$ , can A and B be independent?  
(Only if  $P(A)=0$  or  $P(B)=1$ )
3. Mr. Jones figures that there is 30% chance that his company will set up a branch office in Phoenix. If it does, he is 60% certain that he will be made manager of this new operation. What is the probability that Jones will be a Phoenix branch office manager?  
(0.18)
4. Suppose that 25% of students at a particular college have SAT scores over 1200. If I pick 5 students at random from this student body, what is the probability that at least one of those picked have SAT scores over 1200?  
(0.763)
5. From a group of 3 freshmen, 4 sophomores, 4 juniors, and 3 seniors a committee of size 4 is randomly selected. Find the probability that the committee will consist of 2 sophomores and 2 juniors.  
(0.036)
6. Let a chip be taken at random from a bowl that contains 6 white chips, 3 red chips, and 1 blue chip. Let the random variable  $X=1$  if the outcome is a white chip; let  $X=5$  if the outcome is a red chip; and let  $X=10$  if the outcome is a blue chip. Find the p.m.f and expected value of X.  
(Ignore this problem)
7. Each of two cabinets identical in appearance has 2 drawers. Cabinet A contains a silver coin in each drawer, and cabinet B contains a silver coin in one of its drawers and a gold coin in the other. A cabinet is randomly selected, one of its drawers is opened, and a silver coin is found. What is the probability that there is a silver coin

in the other drawer?  
(0.667)

8. Two cards are drawn successively and without replacement from an ordinary deck of playing cards. Compute the probability of drawing:
  - (a) A heart on the first draw, a club on the second draw.  
(0.064)
  - (b) Are the events  $A$  =heart on the first draw and  $B$  =club on the second draw independent? Rigorously justify your answer.  
(No)
  - (c) A heart on the first draw, an ace on the second draw.  
(0.0192)
9. A bin contains 5 defective (that immediately fail when put in use), 10 partially defective (that fail after a couple of hours of use), and 25 acceptable transistors. A transistor is chosen at random from the bin and put into use. If it does not immediately fail, what is the probability it is acceptable?  
(25/35)
10. A pair of dice are rolled. Let  $E$  denote the event that the sum of the two dice is equal to 7 and  $F$  denote the event that the first die lands on 4. Is  $E$  independent of  $F$ ? Rigorously justify your answer.  
(Yes)
11. Probability of having a toothache because of a cavity is .8. Suppose the probability of having a cavity is .05 in general. Assuming the probability of toothache given you have no cavity is .01, calculate the probability of a cavity given you have a toothache.  
(0.808)
12. A committee consists of five Turks, two Greeks, three Italians, and two Canadians.
  - (a) A subcommittee of four is chosen at random. What is the probability that all the nationalities are represented on the subcommittee?  
(0.1212)
  - (b) Answer the question for part (a) if a subcommittee of five is chosen.  
(0.303)