Stat/Math 414.1 Spring 2008

Review for Final

Our Final will be given on Tuesday, May 6, 10:10AM-12:00PM in 201 Thomas. It will be a closed book and closed notes exam. You are allowed to bring your CALCULATOR and one 8.5 x 11 (letter size) double-sided formula sheet PLUS the two formula sheets used in the midterms with only handwritings on them.

Generally speaking, The final is comprehensive and may cover the whole contents we learned. However, the major part (about 80%) is from the lectures after the 2nd midterm, and the rest (about 20%) is from previous lectures. The exam will include 6-8 problems close to the homework and/or the examples given in the class. In order to get full credit, You are required to show your work, not just give the numerical answer.

1. Normal Probability Distribution, z-score and Normal Table. Note that $P(Z > z_\alpha) = \alpha$; $P(a < X < b) = P(\frac{a-\mu}{\sigma} < Z < \frac{b-\mu}{\sigma})$.
2. Gamma Probability Distribution, the form of general gamma pdf, gamma function and properties, relationship between exponential distribution, χ^2 distribution and gamma distribution, their expectation and variance, mgf of gamma function.
3. Beta Probability Distribution, beta pdf, expectation and variance, beta function, $B(\alpha, \beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$.
4. Use Tchebysheff's Theorem to estimate probabilities.
5. Concepts of Joint Distribution Function and Density Function, Marginal and Conditional Probability Distribution and Density. For continuous case, $f_X(x) = \int f(x,y)dy$, $f_Y(y) = \int f(x,y)dx$, $f_{X|Y=y}(x) = \frac{f(x,y)}{f_Y(y)}$, $f_{Y|X=x}(y) = \frac{f(x,y)}{f_X(x)}$. **Remember to clearly write down the domain of any pdf, cdf or pmf you find.**

6. Compute Probabilities Based on Bivariate Density Functions/Conditional Density Functions, Expected Values of functions of Bivariate r.v.s.

7. How to tell independence of two random variables? Check domain first and then to see if $f(x, y) = g(x)h(y)$ of not.

8. Covariance of Two Random Variables and linear combinations, relationship between variance and covariance, Expected Value and Variance of Linear Combinations of Random Variables. **Know how to use independence to simplify the calculations of expectation, variance and covariance.**

9. Find pdf of transformations of r.v.s, such as $Y = exp(X)$, $Y = X_1 - X_2$, $Y = 3X - X^3$, etc. **Methods:**

   (a) Distribution Function Method

   (b) Method of Transformation

   (c) Method of Moment-Generating Functions

10. Sample Mean of Independent Random Variables, Central Limit Theorem.

11. **You may refer to the previous 2 reviews to go over the contents before the 2nd midterm.**

More practice problems:

Example 1. *The joint density of Y_1 and Y_2 is given by*

$$f(y_1, y_2) = \begin{cases} 30y_1y_2^2, & y_1 - 1 \le y_2 \le 1 - y_1, 0 \le y_1 \le 1 \\ 0, & elsewhere \end{cases}$$

a. Find $F(1/2, 1/2)$.

b. Find $F(1/2, 2)$.

c. Find $P(Y_1 > Y_2)$.

d. Find $P(Y_1 > 1/2 | Y_1 > 1/4)$.

e. Find $P(Y_1 > 1/2 | Y_2 > 1/4)$.

f. Find $f(y_2)$.

g. Find $f_{Y_1|Y_2=y_2}(y_1)$.

h. Find $P(Y_1 > 1/2 | Y_2 = 1/2)$.

i. Find $E(2Y_1 - 3Y_2)$.

j. Find $E(2Y_1 | Y_2 = 0.5)$.

Example 2. *Let Y be a random variable with density function*

$$f(y) = \begin{cases} 3/2y^2, & -1 \le y \le 1 \\ 0, & \textit{elsewhere} \end{cases}$$

a. Find the density function of $U = 3Y$.

b. Find the density function of $U = 3 - Y$.

c. Find the density function of $U = Y^3$.

d. Find the density function of $U = Y^2$.

Example 3. *Assume that test scores of all high school seniors have mean $\mu = 60$ and variance $\sigma^2 = 64$. A random sample of $n = 100$ students from one large high school had a mean score of 58. Is there evidence to suggest that this high school is inferior? (Calculate the probability that the sample mean is at most 58 when $n = 100$.)*

Solution: Let $\bar{Y}$ be the mean of a random sample of $n = 100$ scores from a population with $\mu = 60$ and $\sigma^2 = 64$. We want to find $P(\bar{Y} \le 58)$. By Central Limit Theorem, $\sqrt{n}(\bar{Y} - \mu)/\sigma$ is approximately a standard normal random variable, which we denote by Z. Therefore,

$$\begin{aligned} P(\bar{Y} \le 58) &= P\Big(\frac{\sqrt{n}(\bar{Y} - \mu)}{\sigma} \le \frac{\sqrt{n}(58 - \mu)}{\sigma}\Big) \\ &\approx P\Big(Z \le \frac{\sqrt{n}(58 - \mu)}{\sigma}\Big) \\ &= P\Big(Z \le \frac{\sqrt{100}(58 - 60)}{\sqrt{64}}\Big) = P(Z \le -2.5) = 0.0062. \end{aligned}$$

Because this probability is so small, it is unlikely that the sample from the school of interest can be regarded as a random sample from a population with $\mu = 60$ and $\sigma^2 = 64$. The evidence suggests that the average score for this high school is lower than the overall average of $\mu = 60$.

Q: Find the probability that the sample average of a random sample of $n' = 64$ *students from all high school seniors will be between 56 and 62. Without computing the actual probability, how do you access the chance of getting a sample average above 70?*