4.47 a. The procedure is reversed now, because the area under the curve is known. The objective is to determine the particular values, $z_0$, that will yield the given probability. In this exercise it is necessary to find a $z_0$ such that $P(Z > z_0) = .5000$. By the symmetry of the normal distribution, half of the area falls on each side of the mean. Thus, $P(Z > 0) = .5000$ and the desired value of $z_0$ is 0.

b. A value of $z_0$ is desired such that $P(Z < z_0) = .8643$. Thus the probability, .8643, will be the entire area under the curve to the left of the value $z = z_0$. Notice that the probability is greater than .5, so that $z_0$ must be in the right-hand half of the curve (i.e., $z_0 > 0$). See Figure 4.15. Then

$P(Z < z_0) = 1 - A(z_0) = .8643$
$A(z_0) = .1357$

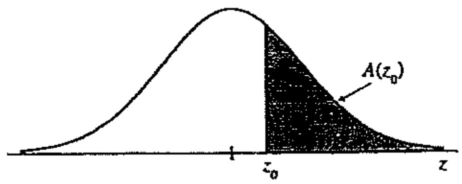


Figure 4.15

From Table 4 we get $z_0 = 1.10$. If the exact probability cannot be found in the table, we may choose to search for the probability closest to the one desired and perform an interpolation that will determine the exact value of $z_0$.

c. It is given that $P(-z_0 < Z < z_0) = .9000$. See Figure 4.16. That is,

$A_1 + A_2 = .9000 = 1 - 2A(z_0)$
$2A(z_0) = .1$
$A(z_0) = .05$

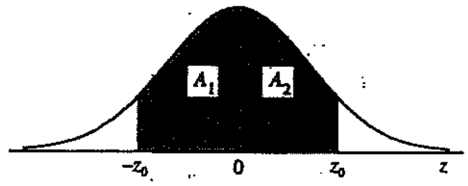


Figure 4.16

The desired value is not tabulated in Table 4 but falls between two tabulated values, .0505 and .0495. Hence $z_0$ will lie between 1.64 and 1.65, which are the $z$ values associated with the above probabilities. Intuitively, one can see that the value .05 is halfway between the two tabulated values, and thus the desired value of $z_0$ will be halfway between 1.64 and 1.65, or $z_0 = 1.645$. This method of evaluation is called "linear interpolation."

d. Similar to part c. In this case,

$P(-z_0 < Z < z_0) = 1 - 2A(z_0) = .9900$

so that $A(z_0) = .0050$. Linear interpolation may now be used to determine the value of $z_0$, which will be between $z_1 = 2.57$ and $z_2 = 2.58$. Hence

$z = 2.57 + \frac{.0050-.0049}{.0051-.0049}(2.58 - 2.57) = 2.57 + \frac{.0001}{.0002}(.01) = 2.57 + .005 = 2.575$

4.57 Let $Y$ be the measured resistance of a randomly selected wire.

a. The required probability is

$$P(.12 \le Y \le .14) = P\left(\frac{.12-.13}{.005} \le \frac{Y-\mu}{\sigma} \le \frac{.14-.13}{.005}\right)$$
$$= P(-2 \le Z \le 2) = 1 - 2P(Z > 2) = 1 - 2A(2)$$
$$= 1 - 2(.0228) = .9544$$

b. Define $X$ to be the number of wires that meet the specifications. Then $X$ has a Binomial distribution with $n = 4$ and $p = .9544$. Thus,

$P(\text{all four will meet the specifications}) = P(X = 4) = \binom{4}{4}(.9544)^4(.0456)^0 = .83$

4.59 a. The $z$ values corresponding to $y_1 = 947$ and $y_2 = 958$ are

$$z_1 = \frac{947-950}{10} = -.3 \qquad \text{and} \qquad z_2 = \frac{958-950}{10} = .8$$

Then

$$P(947 \le Y \le 958) = P(-.3 \le Z \le .8) = 1 - P(Z < -.3) - P(Z > .8)$$
$$= 1 - .3821 - .2119 = .406$$

b. It is necessary that $P(Y \le C) = .8531$. Apparently, then, $C$ must be to the right of the mean, $\mu = 950$, and must be associated with a $z$ value such that

$$A(z) = A\left(\frac{C-950}{10}\right) = .1469$$

This value of $z$ is $z = 1.05$ (from Table 4). Hence

$$\frac{C-950}{10} = 1.05 \qquad \text{and} \qquad C = 960.5$$

**4.61** It is given that the random variable $Y$ (ounces of fill) is normally distributed with mean $\mu$ and standard deviation $\sigma = .3$. The objective is to find a value of $\mu$ so that $P(Y > 8) = .01$. That is, an 8-ounce cup will overflow when $Y > 8$, and this should happen only 1% of the time. The $z$ value corresponding to $Y = 8$ is

$$z = \frac{y-\mu}{\sigma} = \frac{8-\mu}{.3}$$

Thus,

$$P(Y > 8) = P\left(Z > \frac{8-\mu}{.3}\right) = .01 \qquad \text{or} \qquad A\left(\frac{8-\mu}{.3}\right) = .01$$

Consider $z_0 = \frac{8-\mu}{.3}$ and determine the value of $z_0$ that satisfies the equality shown above. This value is 2.33. Hence the value for $\mu$ can be obtained as

$$\frac{8-\mu}{.3} = 2.33 \qquad \text{or} \qquad \mu = 7.301$$

**4.69** Let $Y$ = magnitude of earthquake. $Y$ is exponential with $\beta = 2.4$.

a. $P(Y > 3) = \int_3^\infty \left(\frac{1}{2.4}\right) e^{-y/2.4}\, dy = -e^{-y/2.4}\Big]_3^\infty = e^{-3/2.4} = .2865$

b. $P(2 < Y < 3) = \int_2^3 \left(\frac{1}{2.4}\right) e^{-y/2.4}\, dy = -e^{-y/2.4}\Big]_2^3 = .1481$

**4.76** Notice that the variable part of $f(y)$ is that of a gamma-type random variable with $\alpha = 4$ and $\beta = 2$. In order for the density to integrate to 1, $k$ must be the constant that accompanies this density. Hence

$$k = \frac{1}{\beta^\alpha \Gamma(\alpha)} = \frac{1}{\Gamma(4)2^4} = \frac{1}{6(16)} = \frac{1}{96}$$

**4.79** $P(Y > \lambda) = \int_\lambda^\infty \frac{y^{\alpha-1}e^{-y}}{\Gamma(\alpha)}\, dy = \sum_{y=0}^{\alpha-1} \frac{\lambda^y e^{-\lambda}}{y!}$

For $\lambda = 1$ and $\alpha = 2$, this becomes $P(Y > 1) = \sum_{y=0}^{1} \frac{e^{-1}}{y!} = e^{-1} + e^{-1} = 2e^{-1} = .736$.

**4.86** Since $Y$ has a gamma distribution with $\alpha = 1000$ and $\beta = 20$,

$$E(Y) = \alpha\beta = 20{,}000 \qquad \text{and} \qquad V(Y) = \alpha\beta^2 = 400{,}000$$

The standard deviation is $\sigma = \sqrt{400{,}000} = 632.46$. Hence the point $y = 40{,}000$ lies $\frac{40{,}000-20{,}000}{632.46} = 31.62$ standard deviations above the mean. It is highly unlikely that such a value would be observed.