4.94 a. $F(y) = \int_0^y (6t - 6t^2)\,dt = [3t^2 - 2t^3]_0^y = 3y^2 - 2y^3$ for $0 \le y \le 1$. $F(y) = 0$ for $y < 0$; $F(y) = 1$ for $y > 1$.

b. The graphs for $F(y)$ and $f(y)$ are shown in Figures 4.20 and 4.21.

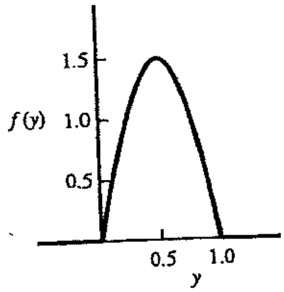

Figure 4.20

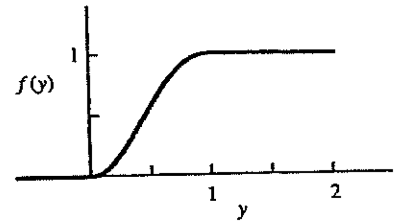

Figure 4.21

c. $P(.5 \le Y \le .8) = F(.8) - F(.5) = 1.92 - 1.024 - .75 + .25 = .396$

4.98 The density function for Y is $f(y) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} y^{\alpha-1}(1-y)^{\beta-1}$ for $0 \le y \le 1$. Thus,

$$E(Y) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \int_0^1 y^{\alpha}(1-y)^{\beta-1}\,dy$$

The quantity $y^{\alpha}(1-y)^{\beta-1}$ is the variable factor of a beta density function with parameters $\alpha + 1$ and β. Hence

$$E(Y) = \frac{\Gamma(\alpha+\beta)\Gamma(\alpha+1)\Gamma(\beta)}{\Gamma(\alpha)\Gamma(\beta)\Gamma(\alpha+\beta+1)} \int_0^1 \frac{y^{\alpha}(1-y)^{\beta-1}}{B(\alpha+1,\beta)}\,dy = \frac{\Gamma(\alpha+\beta)\Gamma(\alpha+1)}{\Gamma(\alpha)\Gamma(\alpha+\beta+1)} = \frac{\alpha}{\alpha+\beta}$$

since the integral of a complete density function is 1. Similarly,

$$E(Y^2) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \int_0^1 y^{\alpha+1}(1-y)^{\beta-1}\,dy = \frac{\Gamma(\alpha+\beta)\Gamma(\alpha+2)\Gamma(\beta)}{\Gamma(\alpha)\Gamma(\beta)\Gamma(\alpha+\beta+2)}$$
$$= \frac{(\alpha+1)\alpha}{(\alpha+\beta)(\alpha+\beta+1)}$$

and

$$V(Y) = \frac{(\alpha+1)\alpha}{(\alpha+\beta)(\alpha+\beta+1)} - \frac{\alpha^2}{(\alpha+\beta)^2} = \frac{(\alpha+1)\alpha(\alpha+\beta) - \alpha^2(\alpha+\beta+1)}{(\alpha+\beta)^2(\alpha+\beta+1)}$$
$$= \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$$

4.99 Let Y = measurement error.

a. $P(Y < .5) = \int_0^{.5} \left[\frac{\Gamma(1+2)}{\Gamma(1)\Gamma(2)}\right] y^{1-1}(1-y)^{2-1}\,dy$

$$= \int_0^{.5} 2(1-y)\,dy = 2y - y^2]_0^{.5} = 1 - .25 = .75$$

b. $\mu = E(Y) = \frac{\alpha}{\alpha+\beta} = \frac{1}{3}$.

$V(Y) = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)} = \frac{2}{3^2(4)} = \frac{1}{18}$; $\sigma = \frac{1}{\sqrt{18}} = .2357$

4.114 The interval must include 90% of all mileage on tires he sells. Using Tchebysheff's theorem, we must have

$$P(|Y - \mu| \le k\sigma) \ge .90 = 1 - \frac{1}{k^2}.$$

Then $k = \sqrt{\frac{1}{.10}} = \sqrt{10} = 3.1622$. The necessary interval is then

$|Y - 25{,}000| \le 3.1622(4000)$ or $12{,}351 \le Y \le 37{,}649$

4.115 It is necessary to have $P(|Y - \mu| \le 1) \ge .75$. Hence, $1 - \frac{1}{k^2} = .75$ and $k = 2$. According to Tchebysheff's inequality, then, $1 = k\sigma$ and $\sigma = \frac{1}{k} = \frac{1}{2}$.

4.122 a. $E(Y) = \nu = 7$ and $V(Y) = 2\nu = 14$.

b. $\sigma = \sqrt{V(Y)} = \sqrt{14} = 3.742$

23 is $\frac{23-7}{3.742} = 4.276$ standard deviations above the mean. It is not likely that Y takes on a value of 23 or more.

5.4 a. Notice that all of the probabilities are at least 0 and sum to 1

b. Note $F(1,2) = P(Y_1 \le 1, Y_2 \le 2) = 1$. The interpretation of this value is that every child in the experiment either survived or didn't and used either 0, 1 or 2 seatbelts.

1. (a) $P(Y=0) = \sum_{i=0}^{2}\left[P(y=0 \mid x=x_i)\, p(x=x_i)\right]$

$= 0.69 \times 0.54 + 0.85 \times 0.17 + 0.84 \times 0.29$

$= 0.76$

$P(Y=1) = 1 - P(Y=0) = 0.24$

$P(Y=0 \mid x=0) \cdot P(x=0) = 0.69 \times 0.54 = 0.3726 = P(y=0, x=0)$

$p(y=0) \cdot p(x=0) = 0.76 \times 0.54 = 0.41 \neq P(y=0, x=0)$

$\therefore$ X and Y are not independent.

(b)

$P(X=x, Y=y)$		0	1	2	X
Y	0	0.37	0.14	0.24	
	1	0.16	0.03	0.05	

(c) $P(y=0, x=0) = 0.37 \neq P(y=0) \cdot P(x=0)$

(d) $P(Y=0) = 0.76$

$P(Y=1) = 0.24$

(e) P(X=0|Y=1)=P(X=0,Y=1)/P(Y=1)=0.16/0.24
P(X=1|Y=1)=P(X=1,Y=1)/P(Y=1)=0.03/0.24
P(X=2|Y=1)=P(X=2,Y=1)/P(Y=1)=0.05/0.24

3. (a) $P(X=0) = 0.7$, $P(X=1) = 0.3$

(b) $P(Y=0 \mid X=0) = 6/9$, $P(Y=1 \mid X=0) = 1/3$

$P(Y=1 \mid X=1) = 2/9$, $P(Y=0 \mid X=1) = 7/9$

(c) $P(Y=0, X=0) = \frac{2}{3} \times 0.7 = 0.467$

$P(Y=1, X=0) = \frac{1}{3} \times 0.7 = 0.23$

$P(Y=1, X=1) = \frac{2}{9} \times 0.3 = 0.067$

$P(Y=0, X=1) = \frac{7}{9} \times 0.3 = 0.23$

(d) $P(Y=0) = 0.47 + 0.23 = 0.7$

$P(Y=1) = 0.23 + 0.07 = 0.3$

the same as X