**2.90** Let $A\overline{A}$ = {positive reading for truthful person, negative reading for liar}.

Then the sample space is

$S = \{AA, A\overline{A}, \overline{A}A, \overline{A}\,\overline{A}\}$.

$P(AA) = .10 \times .95 = .095$

$P(A\overline{A}) = .10 \times .05 = .005$

$P(\overline{A}A) = .9 \times .95 = .855$

$P(\overline{A}\,\overline{A}) = .9 \times .05 = .045$

a. $P(AA) = .095$

b. $P(\overline{A}A) = .855$

c. $P(A\overline{A}) = .005$

d. $1 - P(\overline{A}\,\overline{A}) = .955$.

**2.96** Let D denote defective and G denote good.

a. The event of interest is the union of the following three mutually exclusive events:

$DGGD \qquad GDGD \qquad GGDD$

Note that the last defective must be found on the fourth test, but the other may be found on test 1, 2, or 3.

Consider the first event. A defective must be drawn on the first test. This occurs with probability $\frac{2}{6}$. A nondefective must be drawn on each of the next two tests; the probabilities are $\frac{4}{5}$ and $\frac{3}{4}$, respectively. A defective must be drawn on the fourth test. This happens with probability $\frac{1}{3}$. The probability of this intersection is

$$\left(\tfrac{2}{6}\right)\left(\tfrac{4}{5}\right)\left(\tfrac{3}{4}\right)\left(\tfrac{1}{3}\right).$$

The probabilities associated with the other two events are identical to that of the first. Hence, applying the additive law of probability, we obtain the desired probability:

$$3 \times \tfrac{2}{6} \times \tfrac{4}{5} \times \tfrac{3}{4} \times \tfrac{1}{3} = 3 \times \tfrac{4\cdot3\cdot2\cdot1}{6\cdot5\cdot4\cdot3} = \tfrac{1}{5}.$$

b. We must locate the second defective refrigerator on the second, third, or fourth test. Call this event $A$. Consider the following events:

$A_1$: the second defective is found on the second test

$A_2$: the second defective is found on the third test

$A_3$: the second defective is found on the fourth test

Then $A = A_1 \cup A_2 \cup A_3$ is the union of three mutually exclusive events and

$$P(A) = P(A_1) + P(A_2) + P(A_3)$$

$P(A_3) = \frac{1}{5}$ was found in part a. The event $A_1 = DD$ is the only way to obtain the second defective on the second test, and

$$P(A_1) = P(DD) = \tfrac{2}{6} \times \tfrac{1}{5} = \tfrac{1}{15}$$

$A_2$ is the union of the two mutually exclusive events, $DGD$ or $GDD$, which occur with equal probabilities

$$P(DGD) = P(GDD) = \tfrac{4}{6} \times \tfrac{2}{5} \times \tfrac{1}{4} = \tfrac{1}{15} \qquad \text{so} \qquad P(A_2) = \tfrac{2}{15}$$

Thus,

$$P(A) = P(A_1) + P(A_2) + P(A_3) = \tfrac{1}{15} + \tfrac{2}{15} + \tfrac{1}{5} = \tfrac{2}{5}$$

c. One of the two defectives has been found in the first two tests. Thus there are three nondefectives and one defective remaining. The other defective must be found on the third or fourth test. Call this event $B$, and express it as the union of two mutually exclusive events defined below:

$B_1$: the second defective is found on the third test

$B_2$: the second defective is found on the fourth test

Now the probability of event $B_1$ is $P(B_1) = \frac{1}{4}$. Also, the probability of event $B_2$, which is the intersection of a nondefective on the third draw and a defective on the fourth, is $P(B_2) = \frac{3}{4} \times \frac{1}{3} = \frac{1}{4}$. The event of interest is

$$P(B) = P(B_1 \cup B_2) = P(B_1) + P(B_2) = \tfrac{1}{4} + \tfrac{1}{4} = \tfrac{1}{2}.$$

**2.100** Let $R$ denote the event that the specimen turns red and $N$ denote the event that the specimen contains nitrates.

a. Use the law of total probability to obtain

$$P(R) = P(R|N)P(N) + P(R|\overline{N})P(\overline{N}) = (0.95)(0.3) + (0.10)(0.70) = 0.355$$

b. Use Bayes's rule and part a to obtain

$$P(N|R) = \frac{P(N \cap R)}{P(R)} = \frac{P(R|N)P(N)}{P(R)} = \frac{(0.95)(0.3)}{0.355} = 0.803$$

**2.104** Define these events:

$C$: contract lung cancer $S$: worked in a shipyard

Then $P(S|C) = .22$ and $P(S|\overline{C}) = .14$. Also, $P(C) = .0004$. Using Bayes's Rule,

$$P(C|S) = \frac{P(S|C)P(C)}{P(S|C)P(C)+P(S|\overline{C})P(\overline{C})} = \frac{(.22)(.0004)}{(.22)(.0004)+(.14)(.9996)} = \frac{.000088}{.140032}$$
$$= .0006$$

**2.108** Let $F_i$ be the event that the plane is found in region $i$ when it is searched and $N_i$ the event that it is not found in region $i$. Also, let $R_i$ be the event that the plane is in region $i$. Then

$$P(F_i|R_i) = 1 - \alpha_i$$

and further $P(R_i) = \frac{1}{3}$ for all $i$.

a.
$$P(R_1|N_1) = \frac{P(N_1|R_1)P(R_1)}{P(N_1)}$$
$$= \frac{P(N_1|R_1)P(R_1)}{P(N_1|R_1)P(R_1) + P(N_1|R_2)P(R_2) + P(N_1|R_3)P(R_3)}$$
$$= \frac{\alpha_1\frac{1}{3}}{\alpha_1\frac{1}{3}+\frac{1}{3}+\frac{1}{3}} = \frac{\alpha_1}{\alpha_1+2}$$

b. Similarly, $P(R_2|N_1) = \frac{\frac{1}{3}}{\alpha_1\frac{1}{3}+2\frac{1}{3}} = \frac{1}{\alpha_1+2}$

c. $P(R_3|N_1) = \frac{\frac{1}{3}}{\alpha_1\frac{1}{3}+2\frac{1}{3}} = \frac{1}{\alpha_1+2}$

**2.146** Define the following events:

$E$: person is exposed to the flu $F$: person contracts the flu

Consider two employees, one of whom is inoculated and one not. The probability of interest is the probability that at least one contracts the flu. It is convenient to consider the complement of this even. Then

$P$(at least one contracts the flu) $= 1 - P$(inoculated does not contract flu and noninoculated does not contract flu).

Consider the inoculated employee. For this case,

$$P(\overline{F}) = P(\overline{F}E) + P(\overline{F}\,\overline{E}) = P(\overline{F}|E)P(E) + P(\overline{F}|\overline{E})P(\overline{E})$$
$$= (.8)(.6) + (1)(.4) = .48 + .4 = .88$$

Now consider the noninoculated employee. Again,

$$P(\overline{F}) = P(\overline{F}|E)P(E) + P(\overline{F}|\overline{E})P(\overline{E})$$

However, $P(\overline{F}|E) = .1$ instead of .8. Hence

$$P(\overline{F}) = (.1)(.6) + (1)(.4) = .46$$

Thus,

$P$(neither contracts flu) $= (.88)(.46)$

$P$(at least one contracts flu) $= 1 - (.88)(.46) = 1 - .4048 = .5952$

**3.4** Define the following events:

$A$: valve 1 fails $B$: valve 2 fails $C$: valve 3 fails

Notice :

$P(Y = 2) = P(\overline{A} \cap \overline{B} \cap \overline{C}) = .8^3 = .512,$

$P(Y = 0) = P(A \cap (B \cup C)) = P(A)P(B \cup C) = .2(.2 + .2 - (.2)^2) = .072,$

(by the law of total probability) $P(Y = 1) = 1 - .512 - .072 = .416.$

**3.8** Let $R$ denote the event that a rental occurs on a given day; let $N$ denote no rental. Then $P(R) = \frac{1}{5} = .2$. The variable in question is related to a sequence $RNN\ldots NR$, where the number of $N$'s equals the value of the random variable $Y$. Consider the position immediately following the first $R$. This position is filled by an $R$ with probability .2 and by an $N$ with probability .8. The same probabilities hold for the other positions as well. Note that each sequence under consideration must begin with an $R$. Thus,

$P(Y = 0) = P(RR) = .2^2$

$P(Y = 1) = P(RNR) = (.8)(.2)^2$

$P(Y = 2) = P(RNNR) = (.8)^2(.2)^2$

In general,

$P(Y = y) = (.8)^y(.2)^2$

**3.17** Define $G$ to be the gain to a person in drawing one card. $G$ can take on only three values, \$15, \$5, or \$−4, with probabilities as shown in the accompanying table.

| $G$ | $p(G)$ |
|---|---|
| 15 | $\frac{2}{13}$ |
| 5 | $\frac{2}{13}$ |
| −4 | $\frac{9}{13}$ |

Then $E(G) = \Sigma G p(G) = 15\left(\frac{2}{13}\right) + 5\left(\frac{2}{13}\right) - 4\left(\frac{9}{13}\right) = \frac{4}{13} = .31$ ⇒ The expected gain is \$.31