3.10 $E(Y) = \Sigma y p(y) = 1(.4) + 2(.3) + 3(.2) + 4(.1) = 2.0$

$E\left(\frac{1}{Y}\right) = \Sigma \frac{1}{y} p(y) = 1(.4) + \frac{1}{2}(.3) + \frac{1}{3}(.2) + \frac{1}{4}(.1) = .6417$

$E\left(Y^2 - 1\right) = E\left(Y^2\right) - 1 = [1(.4) + 4(.3) + 9(.2) + 16(.1)] - 1 = 5 - 1 = 4$

Using Theorem 3.6,

$$V(Y) = E\left(Y^2\right) - [E(Y)]^2 = 5 - (2)^2 = 1$$

3.13 Let P be a random variable representing the company's profit. Then $P = C - 15$ with probability 98/100 (when A does not occur) and $P = C - 15 - 1000$ with probability 2/100 (when A occurs). Then $E(P) = (C - 15)\frac{98}{100} + (C - 15 - 1000)\frac{2}{100} = 50$. Simplifying we have $C - 15 - 20 = 50$. Thus $C = \$85$.

3.20 Let Y be daily sales. Then Y can take on three possible values, $Y = 0$, 50,000, or 100,000. The value $Y = 0$ will occur if the salesperson contacts either one or two customers and fails to make a sale. Then

$$\begin{aligned} P(Y = 0) &= P(\text{contact one, fail to sell}) + P(\text{contact two, fail to sell}) \\ &= P(\text{contact one}) \cdot P(\text{fail to sell}) \\ &\quad + P(\text{contact two}) \cdot P(\text{fail with 1st}) \cdot P(\text{fail with 2nd}) \\ &= \left(\tfrac{1}{3}\right)\left(\tfrac{9}{10}\right) + \left(\tfrac{2}{3}\right)\left(\tfrac{9}{10}\right)\left(\tfrac{9}{10}\right) = \tfrac{9}{30} + \tfrac{162}{300} = \tfrac{252}{300} \end{aligned}$$

Similarly,

$$\begin{aligned} P(Y = 50{,}000) &= P(\text{contact one, sell}) + P(\text{contact two, sell to one}) \\ &= P(\text{contact one, sell}) + P(\text{contact two, sell to 1st only}) \\ &\quad + P(\text{contact two, sell to 2nd only}) \\ &= \left(\tfrac{1}{3}\right)\left(\tfrac{1}{10}\right) + \left(\tfrac{2}{3}\right)\left(\tfrac{1}{10}\right)\left(\tfrac{9}{10}\right) + \left(\tfrac{2}{3}\right)\left(\tfrac{9}{10}\right)\left(\tfrac{1}{10}\right) = \tfrac{46}{300} \end{aligned}$$

Finally,

$$P(Y = 100{,}000) = P(\text{contact two, sell to both}) = \left(\tfrac{2}{3}\right)\left(\tfrac{1}{10}\right)\left(\tfrac{1}{10}\right) = \tfrac{2}{300}$$

Then

$$E(Y) = \sum_y y p(y) = 0\left(\tfrac{252}{300}\right) + 50{,}000\left(\tfrac{46}{300}\right) + 100{,}000\left(\tfrac{2}{300}\right) = \tfrac{25{,}000}{3} = 8333$$

Thus, the expected value of daily sales is \$8333.

3.23 a. We let $g_1(Y) = aY$ and $g_2(Y) = b$.

Then, by Theorem 3.5,

$$\begin{aligned} E(aY + b) &= E[g_1(Y) + g_2(Y)] \\ &= E[g_1(Y)] + E[g_2(Y)] = E[aY] + E[b]. \end{aligned}$$

We now use Theorems 3.4 and 3.3 to get $E(aY + b) = aE(Y) + E(b) = a\mu + b$.

b. By Definition 3.5 and exercise a.,

$$\begin{aligned} V(aY + b) &= E[aY + b - (a\mu + b)]^2 \\ &= E[aY - a\mu + b - b]^2 = E[a(Y - \mu)]^2 \\ &= E\left[a^2(Y - \mu)^2\right]. \end{aligned}$$

Using Theorem 3.4, this equals $a^2 E(Y - \mu)^2$ which equals $a^2 V(Y) = a^2\sigma^2$, by Definition 3.5.

Problem 1.

(b) $\begin{pmatrix} 0 & 1 & 2 & 3 \\ 0.2 & 0.5 & 0.2 & 0.1 \end{pmatrix}$

(c) $E(X) = \sum x_i * p_i = 1.2$

$Var(X) = E(X^2) - E(X)^2 = 2.2 - 1.2^2 = 0.76$

(d) $E(Y)=E(1200*X)=1200*E(X)=1440$

$Var(Y)=Var(1200*X)=1200^2*Var(X)=1094400$

Problem 2.

(a) S={0,1,2,3,4,5}

(b) $P(X=k)=\binom{5}{k}0.3^k 0.7^{5-k}$,k=0,1,2,3,4,5

(c) E(X)=5*0.3=1.5

(d) Var(X)=5*0.3*0.7=1.05

(e) i) P(Cost>20)=P(X>2)=0.163

ii) E(Cost)=E(9*X)=9*1.5=13.5

iii) $Var(Cost)=Var(9*X)=9^2*1.05=85.05$

Problem 3.

(a) S={0,1,2,3}

(b) $$\begin{pmatrix} 0 & 1 & 2 & 3 \\ \binom{17}{3}\binom{3}{0}/\binom{20}{3} & \left[\binom{17}{2}\binom{3}{1}\right]/\binom{20}{3} & \left[\binom{17}{1}\binom{3}{2}\right]/\binom{20}{3} & \left[\binom{17}{0}\binom{3}{3}\right]/\binom{20}{3} \end{pmatrix}$$

(c) E(X)=0.451

(d) Var(X)=0.3436 Std(X)=0.586