3.34 The binomial probability distribution for $n = 5$ is given as

$$p(y) = \binom{5}{y} p^y q^{5-y} \qquad y = 0, 1, 2, 3, 4, 5$$

Substituting the values $p = .1$, $p = .5$, and $p = .9$ in the above formula, the three probability distributions are obtained.

	$p = .1$	$p = .5$	$p = .9$
y	$p(y)$	$p(y)$	$p(y)$
0	.59049	.03125	.00001
1	.32805	.15625	.00045
2	.07290	.31250	.00810
3	.00810	.31250	.07290
4	.00045	.15625	.32805
5	.00001	.03125	.59049

Note that when $p = .5$, the distribution is symmetric; that is, $p(y) = p(5 - y)$ for $y = 0, 1, 2, 3, 4, 5$. When $p > .5$, the distribution is skewed to the right, and when $p < .5$, the distribution is skewed to the left. The probability distributions for $p = p_0$ and $p = 1 - p_0$ are "mirror images" of each other.

Note that Table 1, Appendix III, could have been used in this exercise.

3.40 The random variable of interest is Y, the number of successful explorations in $n = 10$ explorations. Then Y has a binomial distribution with $p = .1$. Hence

$E(Y) = np = 10(.1) = 1 \qquad V(Y) = npq = 10(.1)(.9) = .9$

3.42 The random variable Y is binomial with $n = 4$, $p = .1$. Hence

$$E(Y) = np = .4$$

and

$$E(Y^2) = V(Y) + [E(Y)]^2 = npq + n^2p^2 = 4(.1)(.9) + (.4)^2 = .52.$$

Then $E(C) = 3E(Y^2) + E(Y) + 2 = 3(.52) + .4 + 2 = 3.96$.

3.44 Let Y = # of fish that survive. Y is binomial with $n = 20$ and $p = 0.8$.

a. $P(Y = 14) = P(Y \le 14) - P(Y \le 13) = .196 - .087 = .109$

b. $P(Y \ge 10) = 1 - P(Y \le 9) = 1 - .001 = .999$

c. $P(Y \le 16) = .589$

d. $\mu = np = 20(.8) = 16$; $\sigma^2 = npq = 20(.8)(.2) = 3.2$

3.46 **a.** By the multiplication rule!

b. Let Y = # of planes with wing cracks that are detected. Y has a binomial distribution with $n = 3$ and $p = (.9)(.8)(.5) = .36$. $P(Y \ge 1) = 1 - P(Y = 0)$ $= 1 - \binom{3}{0}(.36)^0(.64)^3 = .737856$

3.88 **a.** $P(Y = 1) = \frac{\binom{4}{1}\binom{2}{1}}{\binom{6}{2}} = \frac{(6)(2)}{20} = \frac{3}{5}$

b. $P(Y \ge 1) = p(1) + p(2) = \frac{3}{5} + \frac{\binom{4}{1}\binom{2}{2}}{\binom{6}{3}} = \frac{3}{5} + \frac{1}{5} = \frac{4}{5}$

c. $P(Y \le 1) = p(0) + p(1) = \frac{\binom{4}{3}\binom{2}{0}}{\binom{6}{3}} + \frac{3}{5} = \frac{1}{5} + \frac{3}{5} = \frac{4}{5}$

3.91 The probability of an event as rare or rarer than the one observed can be calculated by using the hypergeometric distribution.

$$P(\text{one or fewer black members}) = \frac{\binom{8}{1}\binom{12}{5}}{\binom{20}{6}} + \frac{\binom{8}{0}\binom{12}{6}}{\binom{20}{6}} = \frac{8(792)}{38{,}760} + \frac{924}{38{,}760} = .187$$

This is not a very unlikely event, since it has probability close to $\frac{1}{5}$. It could very well have happened by chance. There is little reason to doubt the randomness of the selection.

3.150
$$P(Y > 1 | Y \ge 1) = \frac{P(Y>1 \cap Y \ge 1)}{P(Y \ge 1)} = \frac{P(Y>1)}{P(Y\ge 1)} = \frac{1-[P(Y=0)+P(Y=1)]}{1-P(Y=0)} = \frac{1-(1-p)^n - np(1-p)^{n-1}}{1-(1-p)^n}$$

3.172 It is necessary to show that

$$\lim_{N\to\infty} \frac{\binom{r}{y}\binom{N-r}{n-y}}{\binom{N}{n}} = \binom{n}{y} p^y q^{n-y}$$

where $p = \frac{r}{N}$. Consider

$$\frac{\binom{r}{y}\binom{N-r}{n-y}}{\binom{N}{n}} = \frac{r(r-1)\cdots(r-y+1)(N-r)(N-r-1)\cdots(N-r-n+y+1)n(n-1)\cdots(2)(1)}{y(y-1)\cdots(2)(1)(n-y)(n-y-1)\cdots(2)(1)N(N-1)\cdots(N-n+1)}$$

$$= \frac{n(n-1)(n-2)\cdots(2)(1)}{y(y-1)\cdots(2)(1)(n-y)(n-y-1)\cdots(2)(1)} \times \frac{r(r-1)\cdots(r-y+1)N(N-r)(N-r+1)\cdots(N-r-n+y+1)}{N(N-1)(N-2)\cdots(N-n+1)}$$

The left-hand fraction is by definition $\binom{n}{y}$. Counting the number of terms in the right-hand fraction, we see that the numerator $y + (n-y) = n$ terms, as does the denominator. Hence the hypergeometric probability can be written as

$$\binom{n}{y}\left(\frac{r}{N}\right)\left(\frac{r-1}{N-1}\right)\left(\frac{r-2}{N-2}\right)\cdots\left(\frac{r-y+1}{N-y+1}\right)\left(\frac{N-r}{N-y}\right)\left(\frac{N-r-1}{N-y-1}\right)\cdots\left(\frac{N-r-n+y+1}{N-n+1}\right) \quad (*)$$

Consider the limit of the first y fractions. Each is of the form $\frac{r-j}{N-j}$.

$$\lim_{N\to\infty} \frac{r-j}{N-j} = \lim_{N\to\infty}\left(\frac{r}{N-j} - \frac{j}{N-j}\right) = \lim_{N\to\infty}\left(\frac{\frac{r}{N}}{1-\frac{j}{N}} - \frac{j}{N-j}\right) = \frac{r}{N} = p$$

(Note that $\frac{r}{N}$ is held constant.)

Each of the last $n-y$ fractions is of the form $\frac{N-r-k}{N-j}$.

$$\begin{aligned}
\lim_{N\to\infty} \frac{N-r-k}{N-j} &= \lim_{N\to\infty}\left(\frac{N}{N-j} - \frac{r}{N-j}\right) \\
&= \lim_{N\to\infty}\left(\frac{1}{1-\frac{j}{N}} - \frac{\frac{r}{N}}{1-\frac{j}{N}} - \frac{k}{N-j}\right) \\
&= 1 - p = q
\end{aligned}$$

Hence taking the limit in (*), we have

$$\lim_{N\to\infty} \frac{\binom{r}{y}\binom{N-r}{n-y}}{\binom{N}{n}} = \binom{n}{y} p^y q^{n-y}$$