3.55 a. $P(Y > a) = \sum_{y=a+1}^{\infty} q^{y-1}p = q^a \sum_{y=a+1}^{\infty} q^{y-a-1}p = q^a \sum_{z=1}^{\infty} q^{z-1}p = q^a$.

(Notice $\sum_{z=1}^{\infty} q^{z-1}p = 1$ by problem 3.44)

b. Using the result of part a,

$$P(Y > a + b | Y > a) = \frac{P(Y>a+b, Y>a)}{P(Y>a)} = \frac{q^{a+b}}{q^a} = q^b = P(Y > b)$$

Let Y represent the time (in years) until failure of an electrical component. Then b. suggest that the probability the component last b or more years is q^b regardless of how long the component has already lasted. That is, the life of the component has no memory of the past.

3.60 The random variable is Y, the number of consumers interviewed before a success occurs, where a success is defined to be the encountering of a customer who prefers brand A. The random variable Y follows the geometric distribution with $p = P(\text{success}) = .60$.

a. $P(Y = 5) = (.4)^{5-1}(.6) = (.4)^4(.6) = .01536$.

b. $P(Y \geq 5) = 1 - P(Y \leq 4) = 1 - (.6) - (.4)(.6) - (.4)^2(.6) - (.4)^3(.6) = .0256$.

Notice an alternative approach is to use the result of exercise 3.55 **a.**

$P(Y \geq 5) = P(Y > 4) = q^4 = .4^4 = .0256$.

3.70 $P(Y^* = y) = P(Y = y + 1) = q^{(y+1)-1}p = q^y p$. Note the range of values Y^* can take is $0, 1, \ldots$ as $Y^* = Y - 1$ and Y takes values 1,2,...

3.72 The probability of finding an employee with positive indications of asbestos will remain relatively constant from trial to trial provided the number of employees is reasonably large. It then makes sense to define Y as the number of the trial on which the third positive indication of asbestos is observed and model Y as having a negative binomial distribution. Then

$P(10 \text{ employees must be tested in order to find 3 positives}) = P(Y = 10)$

$= \binom{9}{2} (.4)^3(.6)^7 = .06$.

3.78 a. Let Y = # of attempts until you complete your call. Y is geometric with $p = .4$.

$P(\text{complete the call on the first try}) = P(Y = 1) = .4$

$P(\text{complete the call on the second try}) = P(Y = 2) = (.4)(.6) = .24$

$P(\text{complete the call on the third try}) = P(Y = 3) = (.4)(.6)^2 = .144$

b. Let Y = # of attempts until both calls are completed. Y is negative binomial with $r = 2, p = .4$.

$$P(Y = 4) = \binom{3}{1} (.4)^2(.6)^2 = .1728$$

3.98 Let Y be the number of customers arriving. Then Y follows a Poisson distribution with $\lambda = 7$. We perform the calculations exactly, however one could just as easily used table 3 appendix III.

a. $P(Y \leq 3) = p(0) + p(1) + p(2) + p(3) = \frac{7^0 e^{-7}}{0!} + \frac{7^1 e^{-7}}{1!} + \frac{7^2 e^{-7}}{2!} + \frac{7^3 e^{-7}}{3!} = .0818$.

b. $P(Y \geq 2) = 1 - P(Y \leq 1) = 1 - \frac{7^0 e^{-7}}{0!} - \frac{7^1 e^{-7}}{1!} = 1 - 8e^{-7} = .9927$.

c. $P(Y = 5) = \frac{7^5 e^{-7}}{5!} = .1277$.