4.4 a. We need to find k such that

$$1 = \int_0^1 ky(1-y)\,dy = \frac{k}{6} \text{ so that } k = 6.$$

b. $P(0.4 \le Y < 1) = \int_{0.4}^{1} 6y(1-y)\,dy = 0.648.$

c. Continuity implies $P(0.4 \le Y < 1) = P(0.4 \le Y \le 1) = 0.94133.$

d. Note that, by definition, $P(Y \le 0.4 | Y \le 0.8) = \frac{P(Y \le .04)}{P(Y \le 0.8)}$

and

$$P(Y \le 0.4) = \int_0^{0.4} 6y(1-y)\,dy = 0.352$$

$$P(Y \le 0.8) = \int_0^{0.8} 6y(1-y)\,dy = 0.896.$$

Thus,

$$P(Y \le 0.4 | Y \le 0.8) = \frac{0.352}{0.896} = 0.393.$$

e. $P(Y < 0.4 | Y < 0.8) = \frac{P(Y<0.4)}{P(Y<0.8)} = \frac{P(Y \le 0.4)}{P(Y \le 0.8)} = 0.393.$

4.8 a. $f(y)$

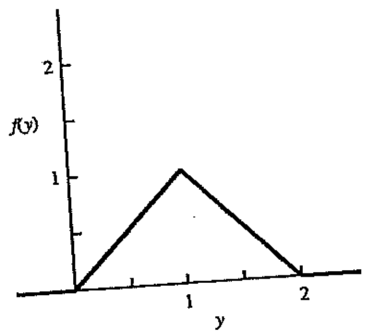

Figure 4.3

b. For $y < 0$, $F(y) = 0$.
For $y > 2$, $F(y) = 1$.
For $0 \le y \le 1$,

$$F(y) = \int_0^y t\,dt = \frac{y^2}{2}$$

For $1 \le y \le 2$,

$$F(y) = \int_0^1 t\,dt + \int_1^y (2-t)\,dt = \frac{1}{2} + \left[2t - \frac{t^2}{2}\right]_1^y = 2y - \frac{y^2}{2} - 1$$

c. $P(.8 \le Y \le 1.2) = F(1.2) - F(.8) = (2.4 - .72 - 1) - .32 = .36$

d. $P(Y > 1.5 \,|\, Y > 1) = \frac{P(Y>1.5)}{P(Y>1)} = \frac{1-(3-1.125-1)}{\frac{1}{2}} = \frac{.125}{.5} = .25$

4.11 a. $F(\infty) = \int_{-\infty}^{\infty} f(y)\,dy = \int_0^1 (cy^2 + y)\,dy = c\left[\frac{y^3}{3}\right]_0^1 + \left[\frac{y^2}{2}\right]_0^1 = \frac{c}{3} + \frac{1}{2} = 1$

Hence $\frac{c}{3} = \frac{1}{2}$ and $c = \frac{3}{2}$.

b. $F(y) = \int_{-\infty}^{y} f(t)\,dt = \int_0^y \left(\frac{3}{2}t^2 + t\right)dt = \frac{t^3}{2}\Big]_0^y + \frac{t^2}{2}\Big]_0^y = \frac{y^3}{2} + \frac{y^2}{2}$ for $0 \le y \le 1$

and $F(y) = 0$ for $y < 0$, $F(y) = 1$ for $y > 1$.

c. The graphs of $F(y)$ and $f(y)$ are shown in Figures 4.6 and 4.7.

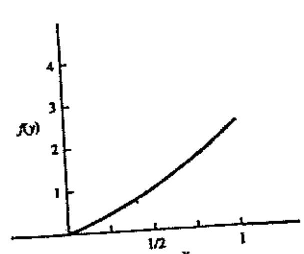

Figure 4.6

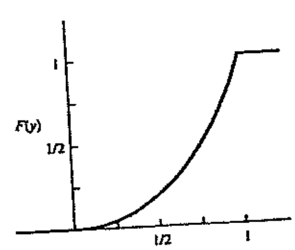

Figure 4.7

d. $F(-1) = 0$ since $y < 0$; $F(0) = 0$; $F(1) = \frac{1}{2} + \frac{1}{2} = 1$

e. $P(0 \le Y \le .5) = F(.5) - F(0) = \frac{(.5)^3}{2} + \frac{(.5)^2}{2} - 0 = \frac{1}{16} + \frac{1}{8} = \frac{3}{16}$

f. $P\left(Y > \frac{1}{2} \middle| Y > \frac{1}{4}\right) = \frac{P\left(Y > \frac{1}{2}\right)}{P\left(Y > \frac{1}{4}\right)} = \frac{1-\left(\frac{3}{16}\right)}{1-\left(\frac{1}{128}+\frac{1}{32}\right)} = \frac{\frac{13}{16}}{\frac{123}{128}} = \frac{104}{123}$

4.13 a. Differentiating $F(y)$ with respect to y, we have

$$f(y) = \begin{cases} 0, & y \le 0 \\ \frac{1}{8}, & 0 < y < 2 \\ \frac{y}{8}, & 2 \le y < 4 \\ 0, & y \ge 4 \end{cases}$$

Notice that $F(y)$ is not differentiable at $y = 0, 2,$ and 4.

b. $P(1 \le Y \le 3) = F(3) - F(1) = \frac{9}{16} - \frac{2}{16} = \frac{7}{16}$

c. $P(Y \ge 1.5) = 1 - F(1.5) = 1 - \frac{1.5}{8} = \frac{13}{16}$

d. $P(Y \ge 1 | Y \le 3) = \frac{P(1 \le Y \le 3)}{P(Y \le 3)} = \frac{\frac{7}{16}}{\frac{9}{16}} = \frac{7}{9}$

4.19 Recall that we found the density function, $f(y)$, in exercise 4.13. Then

$$E(Y) = \int_{-\infty}^{\infty} y f(y)\,dy = \int_0^2 y/8\,dy + \int_2^4 y^2/8\,dy = \left[\frac{y^2}{16}\right]_0^2 + \left[\frac{y^3}{24}\right]_2^4 = 31/12.$$

To find the variance we apply the result of exercise 4.18. That is, we need to calculate

$$E(Y^2) = \int_{-\infty}^{\infty} y^2 f(y)\,dy = \int_0^2 y^2/8\,dy + \int_2^4 y^3/8\,dy = \left[\frac{y^3}{24}\right]_0^2 + \left[\frac{y^4}{32}\right]_2^4 = 47/6.$$

Thus $V(Y) = E(Y^2) - (EY)^2 = 47/6 - (31/12)^2 \approx 7.83 - 6.67 = 1.16.$

4.22 a. To solve for c, we need to consider $\int_0^1 cy^2(1-y)^4\,dy = 1.$

Integrating, we have $c\left(\frac{y^3}{3} - y^4 + \frac{6y^5}{5} - \frac{4y^6}{6} + \frac{y^7}{7}\right)\Big]_0^1 = 1.$

$c\left[\left(\frac{1}{3}\right) - 1 + \left(\frac{6}{5}\right) - \left(\frac{4}{6}\right) + \left(\frac{1}{7}\right)\right] = 1.$

$c = (1)\left(\frac{630}{6}\right) = 105.$

b. $E(Y) = 105\int_0^1 yy^2(1-y)^4\,dy = 105\left[\left(\frac{y^4}{4}\right) + \left(-\frac{4y^5}{5}\right) + y^6 + \left(-\frac{4y^7}{7} + \frac{y^8}{8}\right)\right]_0^1 = \frac{3}{8}.$

4.36 Let the interval be $(0, 500)$ so that the location Y has density $f(y) = 1/500$ for $0 < y < 500$ and 0 otherwise.

a. $P(475 < Y < 500) = \int_{475}^{500} 1/500\,dy = 1/20$

b. $P(0 < Y < 25) = \int_0^{25} 1/500\,dy = 1/20$

c. $P(0 < Y < 250) = \int_0^{250} 1/500\,dy = 1/2$