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Multicriterion Prioritization with Differential Weights, Stepwise Aggregation, Hasse Diagrams, and Poset Cumulative Rank Frequency Operators

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This material is based upon work partially supported by (i) the National Science Foundation under Grant No. 0307010, (ii) The United States Environmental Protection Agency under Grant No. CR-83059301 and (iii) The Pennsylvania Department of Health using Tobacco Settlement Funds and Grant No. ME 01324. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the agencies.

[Presented at the 7th Annual International Conference on Digital Government Research]

Technical Report Number 2006-0532
TECHNICAL REPORTS AND REPRINTS SERIES
May 2006

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Environmental and Ecological Statistics-Springer

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ABSTRACT
There has been considerable work on determining a suitable method to accomplish a satisfactory ordering of a group of objects, when there are multiple evaluation criteria. A weighted index can be used to combine the opinion of all stakeholders to obtain a criterion for ranking the objects. We also use elements of Poset (Partial order set) theory to determine the rankings of the objects. The Poset linear extension method [3] can be used to find rankings without using an index, relying only on pairwise comparisons of the objects. Our goal here is to explore several methods of selecting effective weights using the methods of analysis such as POSAC and METEOR, which will give us insight for improving the weights already selected by the stakeholders.

Categories and Subject Descriptors
H.4 [Information Systems Application]: types of systems

General Terms
Multicriterion decision making, weights, aggregations, posets.

Keywords
Multicriterion prioritization, differential weightings, stepwise aggregation, cumulative rank frequency distribution operators, Atlantic slope consortium, bridge crossings.

1. INTRODUCTION
There are two data sets that we use for further exploration of these methods. One is data for a set of 21 watersheds studied by the Atlantic Slope Consortium (ASC) (see [1]), with the goal of determining an accurate ranking of their condition. This data set has three different levels of indicators, grouped into Level I to Level III, increasing in the quality and accuracy of the data as well as the amount of cost and effort needed to obtain the data. The spatial coverage of the data decreases going from Level I to Level III. We consider the two indicators of Level III to be the best quality of data we have regarding the ecological condition of the watershed. Due to the money and effort in the procedure of obtaining this data, it is available for only six watersheds. We have for all 21 watersheds, data for seven Level II indicators, and for five Level I indicators. The data matrix for Level II indicators has 21 rows and 7 columns. The data were normalized so every value in the data matrix is a “score” between zero and one. In addition, each watershed was identified with one of four physiographic regions, which describes the terrain of the land, and one of six social choices, which describes the use of the land (urban, agriculture, forest, mixed etc.). An index using all seven of the Level II indicators has already been created by the scientists, with the selected weights based on a conceptual model of condition. It is referred to as the Stream-Wetland-Riperian (SWR) Index. This scheme considers a two level hierarchy: four indicators are considered as part of a Floodplain/Wetland (FW) subindex, and then the SWR index is an index of the other three indicators and FW, see Figure 1. The scientists chose to average the indicators for both indices, resulting in an effective weight of 1/16 for the four indicators in the FW subindex and ¼ for the other three indicators.

The second data set we use for this paper is a data set of 49 streams at bridge crossings with the goal of determining the stability of the stream channel and whether it threatens the bridge structure [2]. Here we have 13 indicators, which the investigators group into four different groups, a two-level hierarchy with four subindices. The investigators experimented with two indices, one, which was an average of the all 13 indicators, the other the average of the four subindices, which were in turn the average of the indicators in the respective subindex. Our main goals for this data set are to identify which indicators are most influential in determining the rankings of bridges in susceptibility to bridge failure due to instability of the stream channel that they cross, and ranking of the bridge sites themselves.

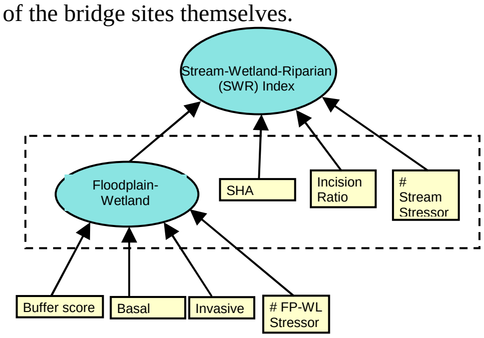

Figure 1 – The Conceptual Model of the SWR Index

METHODS AND RESULTS
In order to understand the reasoning behind the weights and to propose more effective weights, we use step-by-step aggregation using METEOR [4]. The idea behind METEOR is that at each step multiple indicators (usually two) are merged together by a weighted linear combination, i.e. $I=g_1*I_1+(1- g_1)* I_2$. We choose the indicators to merge by choosing the least correlated indicators or by taking advantage of a natural hierarchy among the indicators. The eventual result is equivalent to an index of all the indicators, however the steps in the aggregation give us an idea of the influence of the indicators. Comparabilities of Hassediagrams are used as a metric for comparison between the two methods, two objects are comparable if one object is better than or equal than the other, and we compute the number of such pairs. The aggregation of the two indicators increases the number of comparabilities in the Hassediagram as contradictory rankings between the two indicators are eliminated. When an indicator is aggregated with the set of already aggregated indicators, we compute the difference of number of comparabilities with the last indicator aggregated from the number of comparabilities before aggregation of the last indicator. From this information, one can understand whether a particular indicator is influential or

dissimilar to the other indicators. If the indicator is influential, then it might be worth some effort to question whether the relative weight given that indicator during aggregation is appropriate, and whether the weight should be increased or decreased.

For the ASC watershed data, we applied METEOR for two situations for all 21 watersheds, and for the six different social choices (data was computed by averaging all watersheds in the social choice), see Figure 2. We have a natural hierarchy, with the four indicators that were the part of the FW subindex, and we first aggregate these four indicators step-by-step, at each step aggregating the pair with the lowest correlation. Then the indicators in the higher level of hierarchy can be aggregated in the same manner into one index. In the bridge data, we see a natural hierarchy [2], where the 13 indicators have already been divided into four groups, with indicators 1-4 in group 1, indicators 5-7 in group 2, indicators 8-12 in group 3, and indicator 13 alone in group 4. Aggregation here would also be done in two stages, aggregating the indicators from 13 indicators into the four groups, and then from the four groups into one final index.

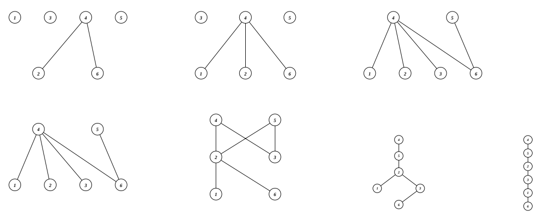

Figure 2 - Step-by-step aggregation for social choices

We want to explore additional methods to determine the selection of weights for an index, using the ASC Level II data set for all 21 watersheds as an example. POSAC (see [5] for more details) is a method generally used to reduce the dimensionality of the indicators, while preserving as many of the comparabilities from the original data matrix as possible. This method is similar to Principal Components Analysis (PCA), however POSAC preserves rank order rather than distances. POSAC plots the data on a two-dimensional plane, and thus produces two latent order variables (LOV). Due to the interest in understanding the strength of the influence of the original indicators on the LOVs, approximate "loadings" are found by computing the F-value of the ANOVA test with the LOV as the response variable and the indicator as the factor. For the ASC Level II data, these F scores were computed for all seven indicators for both LOV variables. To allow for small deviations in the POSAC algorithm, we discretized both the LOVs and the original data into eight equally spaced intervals, and gave an appropriate score, for example, values between 0 and 0.125 would get a score of 1, between 0.125 and 0.25 get a score of 2, etc. We used two different programs, POSAC from SYSTAT and MPOSAC (which allows for more than two dimensions of LOV) from HUDAP.

We used five different methods for obtaining the differential weights for the indicators by using the guidance from the POSAC method as follows. In all the cases we averaged the four values that we obtained from both POSAC and MPOSAC for both LOVs, and normalized the seven values to sum to one, resulting in a set of weights.

1. We used the F-value method described above.
2. The p-values for each of the F-tests in method 1 were computed, and the reciprocal was taken as record values.
3. We computed a "concordance" value for each indicator by counting the number of times that the indicator and LOV gave the same scored value.
4. We computed Spearman's rank correlation between the LOV and the data for the indicator.
5. The average of methods 1-4 for each indicator.

The indices using the above weights can be used to obtain a ranking of the 21 objects, and in turn, a ranking of the six selected watersheds. One performance measure can be the correlation of the ranking obtained by the index with the ranking of the Level III indicators, the gold standard! Table 1 shows the correlation and p-value for each method, as well as for the SWR index. We still are impressed by the concordance weights, giving a statistically significant correlation with the Level III rankings.

The methods above may be characterized as investigative methods, in that the weights are computed from the data alone, and without input from the scientists about the indicator weights. We can also use these methods in addition to the weights already provided by the scientists in hopes of improving the weights already provided. In particular, we may use the information from METEOR to help determine a new set of weights.

Table 1: Correlations of rankings from weight selection methods with Level III rankings

Method	Correlation	p-value
1	0.371	0.471
2	0.086	0.872
3	0.886	0.019
4	0.657	0.156
5	0.486	0.329
SWR	0.943	0.005

ACKNOWLEDGEMENTS

This material is based upon work supported by (i) the NSF Grant No. 0307010, and (ii) the US EPA Grant. Nos. R-828684, CR-83059301, and RD-8324400l. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the agencies. Our thanks are due to Robert Brooks, Mary Easterling, and Jessica Newlin for providing data and sharing insights.

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