Ranks and Semiparametric Efficiency in Time Series models

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Semiparametric time series models typically involve a finite-dimensional parameter of interest $\boldsymbol{\theta} \in \boldsymbol{\Theta} \subseteq \mathbb{R}^k$ and an innovation density f playing the role of a nuisance parameter. Quite often, the submodels corresponding to a fixed value of $\boldsymbol{\theta}$ possess a group structure that induces a maximal invariant σ-field $\mathcal{B}(\boldsymbol{\theta})$ which is generated by the ranks (the signs, the signed ranks, ...) of the residuals associated with the parameter value $\boldsymbol{\theta}$. It is shown that semiparametrically efficient (at given f) distribution-free rank-based inference procedures can generally be constructed from parametrically optimal ones by conditioning on $\mathcal{B}(\boldsymbol{\theta})$. The same procedures, when combined with a suitable estimation of the underlying nuisance density f, yield conditionally distribution-free semiparametrically efficient inference methods, e.g., semiparametrically efficient permutation tests. Remarkably, this is achieved without any explicit tangent space or efficient score computations and without any sample-splitting device. By means of several examples, we show how these results apply in time series models for which rank-based inference or permutation tests seldom have been considered so far.

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