A Functional Central Limit Theorem for the Empirical Estimator of a Semi–Markov Kernel

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Abstract:

Consider an infinite countable set, say E, and an E-valued time–homogeneous semi–Markov process $(Z_t)_{t\in\mathbb{R}_+}$, with semi-Markov kernel $Q(t) = (Q_{ij}(t),\ i,j \in E),\ t \geq 0$, and embedded Markov renewal process $(J_n, S_n)_{n\in\mathbb{N}}$. (J_n) is the E-valued embedded Markov chain of the successive visited states, and (S_n) are the jump times of (Z_t). Define also $X_n := S_n - S_{n-1},\ n \geq 1$, the inter–jump times, and the process $(N(t))_{t\in\mathbb{R}_+}$, which counts the number of jumps of (Z_t) in the time interval $(0,t]$. Let also define $N_i(t)$ the number of visits of (Z_t) to state $i \in E$ up to time t.

The empirical estimator of the semi–Markov kernel, based on the observation $\{Z_s(\omega), 0 \leq s \leq t\}$, is defined by

$$\hat{Q}_{ij}(x,t) := \frac{1}{N_i(t)} \sum_{k=1}^{N(t)} \mathbf{1}_{\{J_{k-1}=i, J_k=j, X_k\leq x\}}, \quad 0 \leq x \leq t,\ i,j \in E. \tag{1}$$

Here we present the invariance principle for the empirical estimator (1) of the semi–Markov kernel and some related results and application. The proofs are performed through a martingale approach.