Inferring Likelihoods and Climate System Characteristics from Climate Models and Multiple Tracers

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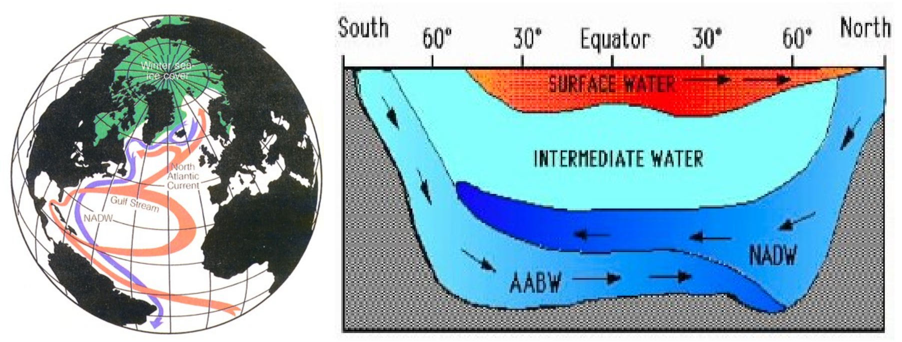
Motivation

- *Example of climate change: potential collapse of meridional overturning circulation (MOC), results in disruptions in the equilibrium state in the climate.*
- *An MOC collapse may result in drastic changes in temperatures and precipitation patterns.*

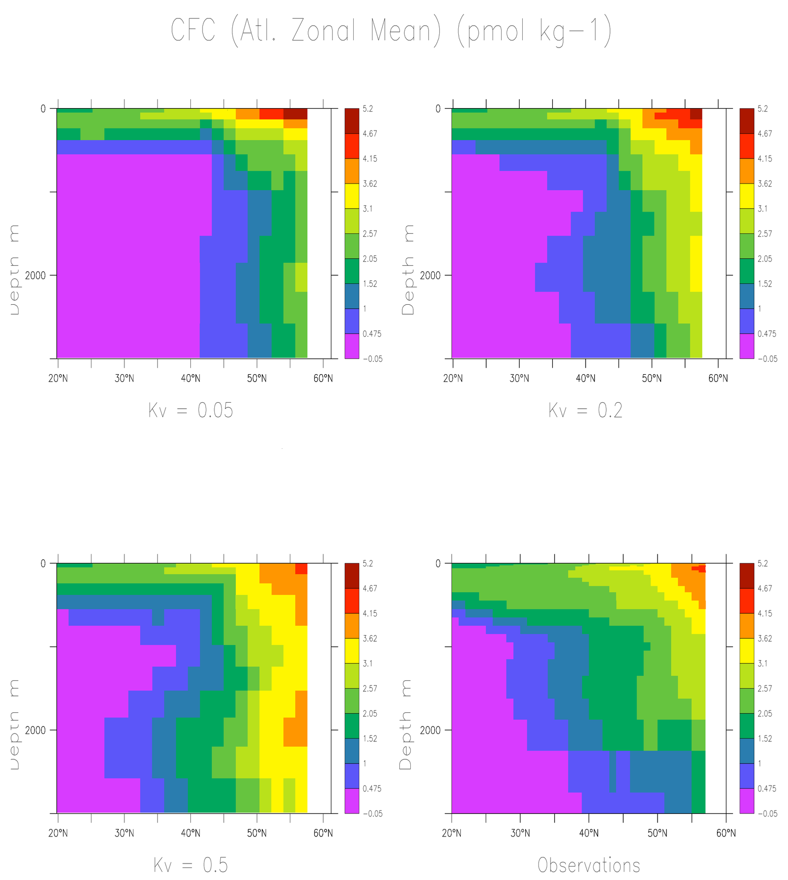


Problem Statement

- **Goal**: Infer important climate characteristics (parameters) that drive major climate systems.
- Sources of information
  - Physical observations of climate system E.g. spatial data on CFC-11 and C-14.
  - Output from complex climate models at several different climate parameters from University of Victoria(UVic) Earth System Climate Model (Weaver et. al. 2001).
- Challenges
  - No direct connection between observations and climate parameter, need to rely on sparse climate model runs.
  - Large data sets: both observations and climate model output.
  - Combining information from multiple tracers (multivariate spatial data).

Tracers

- Predictions of MOC strength can be made for particular climate parameter settings, such as vertical diffusivity,(K_v).
- K_v values cannot be measured directly.
- Trichlorofluoromethane (CFC11) and Carbon-14 (^{14}C) are considered as stable tracers, used to infer K_v, related to deep ocean behavior such as MOC strength.
- Observations of both tracers collected across globe in the 1990s, locations consist of latitude, longitude, and depth values, aggregated over longitudes.
- Second source of information: climate model output at different values of K_v.
- 3706 observations and 5926 data points from model.
- Latitude between -80 S and 60 N, depths from 0 to 3000m.



Bottom right: observations, other plots are model output.

Statistical Inference: Related Work

- Notation: $\boldsymbol{Z}(\mathrm{s})$ are physical observations at location s=(latitude, depth).
- $\boldsymbol{Y}(\mathrm{s}, \boldsymbol{\theta})$ are climate model output at location s and climate parameter $\boldsymbol{\theta}$.
- Kennedy and O'Hagan (2001) developed a model to solve the calibration problem for computer experiments. Sanso et al. (2007) used a variant for climate models.
  - Assumes that a "true" set of climate parameters $\boldsymbol{\theta}^*$ exists.
  - Combine both $\boldsymbol{Y}$ and $\boldsymbol{Z}$ in one model, treat $\boldsymbol{Y}$ as a Gaussian process, with a mean dependent on climate parameter $\boldsymbol{\theta}$:

$$Z(\mathrm{s_i}) = Y(\mathrm{s_i}, \boldsymbol{\theta}^*) + \delta(\mathrm{s_i}) + \epsilon_i$$

  - where δ is the model error term and ϵ is observation error.
  - Separable covariance function between s and $\boldsymbol{\theta}$.

Our Approach

- Two stage approach to obtain posterior of $\boldsymbol{\theta}$: (1) model relationship between $\boldsymbol{Z}$ and $\boldsymbol{\theta}$ via model output $\boldsymbol{Y}$ and (2) Use observations $\boldsymbol{Z}$ to obtain $\boldsymbol{\theta}^*$.
- Model $\boldsymbol{Y}$ as a Gaussian process: $\boldsymbol{Y} \mid \boldsymbol{\beta}, \boldsymbol{\xi} \sim N(\mu_{\boldsymbol{\beta}}(\boldsymbol{\theta}), \Sigma(\boldsymbol{\xi}))$.
- $\boldsymbol{\beta}$: regression parameters, $\boldsymbol{\xi}$:covariance parameters, covariance function separable among s and $\boldsymbol{\theta}$.
- $\boldsymbol{\beta}$ and $\boldsymbol{\xi}$ estimated using maximum likelihood.
- For locations s at a *given* value of $\boldsymbol{\theta}$, we obtain the predictive distribution $\pi(\boldsymbol{Z}^*(\boldsymbol{\theta}) \mid \boldsymbol{Y})$ which is multivariate normal, a function of $\boldsymbol{\theta}$.

$$\boldsymbol{Z} = \hat{\boldsymbol{\eta}}(\boldsymbol{Z}^* \mid \boldsymbol{\theta}^*, \boldsymbol{Y}) + \delta(\mathbf{s}) + \epsilon$$

- where δ is the model error term and ϵ is observation error.
- $\epsilon \sim N(0, \psi I)$ and δ is modeled as a Gaussian process.
- Two stage approach can be seen as a way of 'cutting feedback' (Best et al. 2006; Rougier, 2008).
- Potential advantages (cf. Rougier, 2008):
  - Protecting emulator from a poor model of climate system.
  - Modeling emulator separately to facilitate careful evaluation of emulator.
  - Computational tractability.
  - Identifiability for δ and ϵ, hope to improve our ability to learn about these errors.
- General framework related to Bayarri et al. (2007), and can be applied to single or multiple tracers.
- Potential drawback: lack of unified model.

Bivariate Conditional Hierarchical Model

- How can we combine information from multiple tracers (CFC-11, C-14) to infer K_v?
- Model $(\boldsymbol{Y}_1, \boldsymbol{Y}_2)$ as a hierarchical model: $\boldsymbol{Y}_1|\boldsymbol{Y}_2$ and $\boldsymbol{Y}_2$ as Gaussian processes. (following Royle and Berliner (1999)).

$$\boldsymbol{Y}_1 \mid \boldsymbol{Y}_2, \boldsymbol{\beta}_1, \boldsymbol{\xi}_1, \gamma \sim N(\mu_{\boldsymbol{\beta}_1}(\boldsymbol{\theta}) + \mathbf{B}(\gamma)\boldsymbol{Y}_2, \Sigma_{1.2}(\boldsymbol{\xi}_1))$$
$$\boldsymbol{Y}_2 \mid \boldsymbol{\beta}_2, \boldsymbol{\xi}_2 \sim N(\mu_{\boldsymbol{\beta}_2}(\boldsymbol{\theta}), \Sigma_2(\boldsymbol{\xi}_2))$$

- $\mathbf{B}(\gamma)$ is a matrix to describe relationship between $\boldsymbol{Y}_1$ and $\boldsymbol{Y}_2$, We assume a piecewise linear relationship.

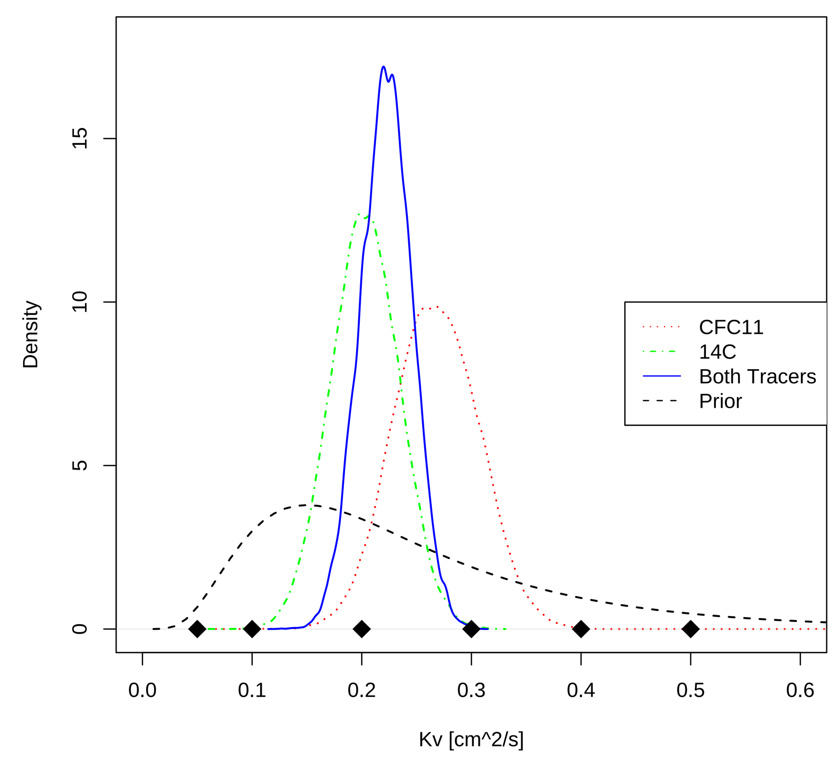
$$\begin{array}{llll}
\mathbf{B}_{ii} = \gamma_1 & \text{if } \ \mathbf{s}_{i2} < 200, \boldsymbol{Y}_{2i} < 2.1 & \mathbf{B}_{ii} = \gamma_2 & \text{if } \ \mathbf{s}_{i2} < 200, \boldsymbol{Y}_{2i} > 2.1 \\
\mathbf{B}_{ii} = \gamma_3 & \text{if } \ 200 < \mathbf{s}_{i2} < 800, \boldsymbol{Y}_{2i} < 1.8 & \mathbf{B}_{ii} = \gamma_4 & \text{if } \ 200 < \mathbf{s}_{i2} < 800, \boldsymbol{Y}_{2i} > 1.8 \\
\mathbf{B}_{ii} = \gamma_5 & \text{if } \ 800 < \mathbf{s}_{i2} < 2500 & \mathbf{B}_{ii} = 0 & \text{if } \ 2500 < \mathbf{s}_{i2} \\
\mathbf{B}_{ij} = 0 & \text{if } \ i \neq j & &
\end{array}$$

$\mathbf{s}_{i2}$ is the depth of the ocean corresponding to the output $(\boldsymbol{Y}_{1i}, \boldsymbol{Y}_{2i})$, and $\boldsymbol{Y}_{2i}$ is the CFC11 value from the climate model.

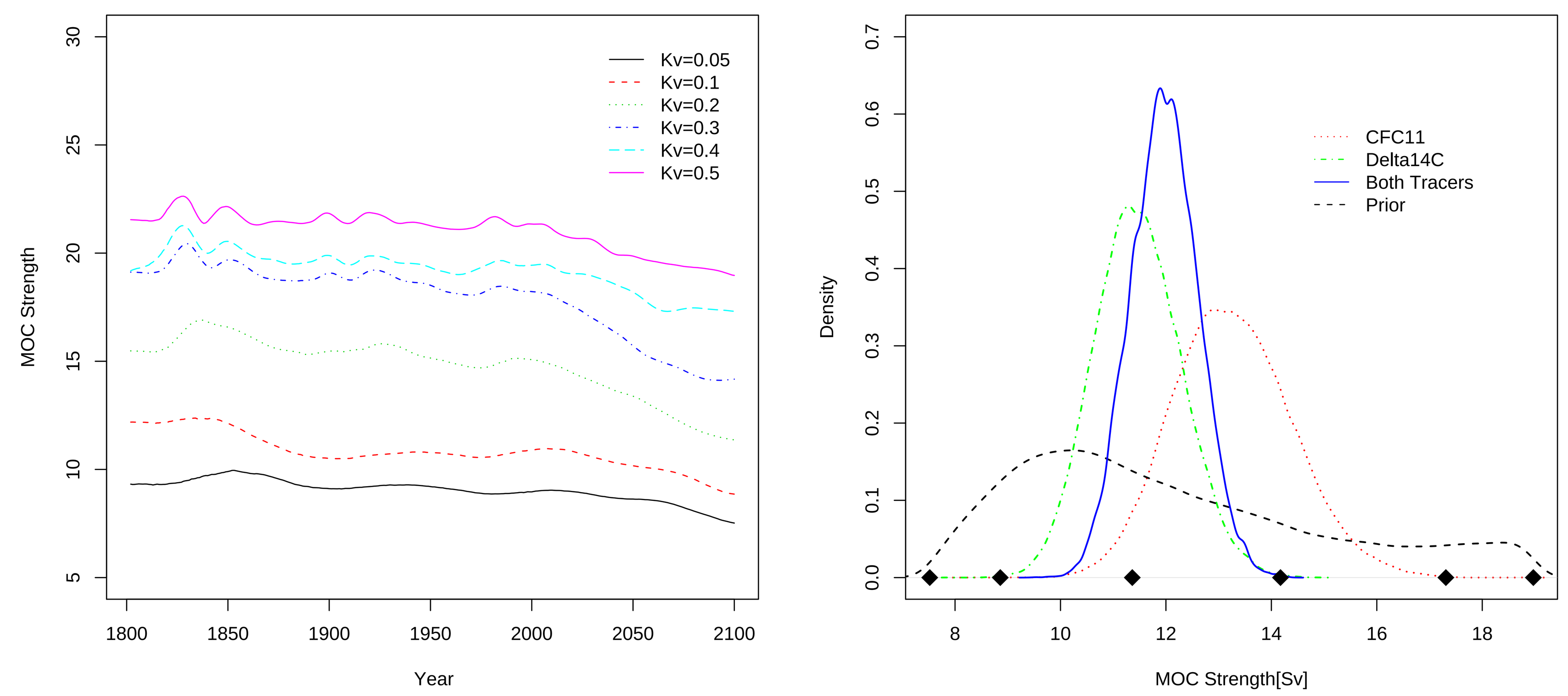
- Likelihoods for $\boldsymbol{Y}_1|\boldsymbol{Y}_2$ and $\boldsymbol{Y}_2$ have no common parameters, and can be maximized separately to obtain MLE.
- Use predictive distribution for $\boldsymbol{Y} = (\boldsymbol{Y}_1\ \boldsymbol{Y}_2)$ to obtain probability model connecting $\boldsymbol{Z} = (\boldsymbol{Z}_1\ \boldsymbol{Z}_2)$ to $\boldsymbol{\theta}$ as before.

Bivariate Model and Computational Issues

- The model for the second stage is $\boldsymbol{Z} = \hat{\boldsymbol{\eta}}(\boldsymbol{Z}^* \mid \boldsymbol{\theta}^*, \boldsymbol{Y}) + \boldsymbol{\epsilon}$ where $\boldsymbol{\epsilon} = (\boldsymbol{\epsilon}_1\ \boldsymbol{\epsilon}_2)^T$ is the observation error, $\boldsymbol{\epsilon}_i = (\epsilon_{1i}, \cdots, \epsilon_{Ni})^T \sim N(0, \psi_{mi} I_N)$, $Cov(\boldsymbol{\epsilon}_1, \boldsymbol{\epsilon}_2) = \mathbf{0}$.
- Computations are $\mathcal{O}(N^3)$, where N is the number of observations (could be tens of thousands).
- Used kernel mixing (Higdon (1998)) and Cressie and Johannesson (2008) to write covariance matrix as: $(\psi I_N + \mathbf{K}(I_J)^{-1}\mathbf{K}^T)$: $\mathbf{K}$ kernel matrix with rank J=196.
- Sherman-Woodbury-Morrison identity used to invert large matrices in above form reduces matrix inversions to $J \times J$, resulting in a computationally tractable model.
- Inference on $\boldsymbol{\theta}^*$,ψ_{m1}, and ψ_{m2} is performed using MCMC, computations simplified by using partitioned matrices.



Conclusion: Our model allows for inference based on *both* tracers jointly, while still remaining computationally tractable.



Left: MOC strength projections in sverdrups for several K_v values between 0.05 and 0.5, where MOC strength increasing as K_v increases. Right: Distribution of projected MOC strength(Sv) in 2100 (left) given posterior distributions of K_v for both single tracers and multiple tracers.

References

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